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Note.—Part 2 of this Journal is the 1961-5 Newsletter of the Soil Mechanics and Foundations Division.

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UNDERPINNING OF A WIND TUNNEL

By James F. McNulty,[1] F. ASCE and James S. O'Brien,[2] M. ASCE

SYNOPSIS

This paper describes the reinforcing of existing pile caps by the addition of new piles which required sectional driving because of space limitations. The project is described from the initial sub-soil investigation through the field construction. An original method is presented for the design of a foundation supported by two types of piles with different stiffnesses and capacities.

INTRODUCTION

A wind tunnel enables the aeronautical engineer to simulate actual flight under laboratory controlled conditions and, as such, is an essential tool for research. Since construction of a wind tunnel is a costly project, often involving 10 to 15 million dollars, consideration is first given to modifying an existing tunnel so that it keeps pace with recent advances. This paper is concerned with foundation problems arising from such a modification.

It was decided to modify the 19-ft pressure tunnel at the Langley Laboratory of the National Advisory Committee of Aeronautics (Langley Field, Virginia) so that it would operate under near vacuum conditions. This entailed a considerable stiffening of the steel tunnel shell which doubled the

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Note.—Discussion open until July 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. SM 1, February, 1961.

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tunnel's previous dead weight of 3,000 tons. Underpinning the structure, with a minimum of working space, to support this additional load was the problem.

DESCRIPTION OF SITE

The wind tunnel was located between two three-story buildings as shown in Fig. 1. The centerline of the tunnel circuit was 33 ft above grade with clearances under the shell varying from 3 ft to 24 ft depending on the diameter of the tunnel at that point. Fig. 2 shows elevation of the tunnel at the large diameter end. In addition, a one-story equipment building occupied most of the open center area between the tunnel legs.

SOILS INVESTIGATION

A complete soils study was made including borings, laboratory soil test and analysis. The results of this study were as follows:

1. The existing 15-ft timber piles were incapable of carrying these increased loads.
2. A high capacity pile should be incorporated with the existing piles for economical design because driving cost would be high in crowded working area.
3. The added piles would have to be driven in sections because of the headroom restrictions. Piles could be located so that there would be a minimum of 17 ft headroom above cut-off and then driven in 8-ft sections with a McKiernan-Terry 9B3 double acting hammer.
4. It was estimated that a 10-3/4 in. pipe pile could be driven 40 ft by a 9B3 without excessively "hard" driving.
5. On the basis of previous pile load tests for projects on Langley Field, failure loads for piles were estimated as 110 tons for the 40-ft pipe pile and 45 tons for the 15-ft timber pile. The stiffnesses, coefficients of vertical pile reaction, for the piles were estimated as 500 kips per in. for the pipe pile and 167 kips per in. for the timber pile. Fig. 3 gives the basis of these estimates.

On the basis of these results, the design criteria were formulated. It was felt that a conservative safety factor was required since (1) nothing was known of the condition of the timber piles driven in 1939, (2) estimates made for failure load of various piles remained to be substantiated by actual field tests and (3) difficulty in working conditions foreshadowed problems of obtaining the usual field workmanship.

The following design criteria were established:

1. Investigate each footing as a unique problem. Decide which of the following alternatives would be the most feasible for each individual footing:

    (a) Incorporate existing cap with timber piles into a new cap with pipe piles.

    (b) Demolish existing cap but incorporate timber piles with pipe piles in a new cap.

    (c) Demolish existing footing and construct new footing with pipe piles.

2. Because caps, in most cases, would include both timber and pipe piles, ordinary design procedures were vitiated. The loads on the piles as well as

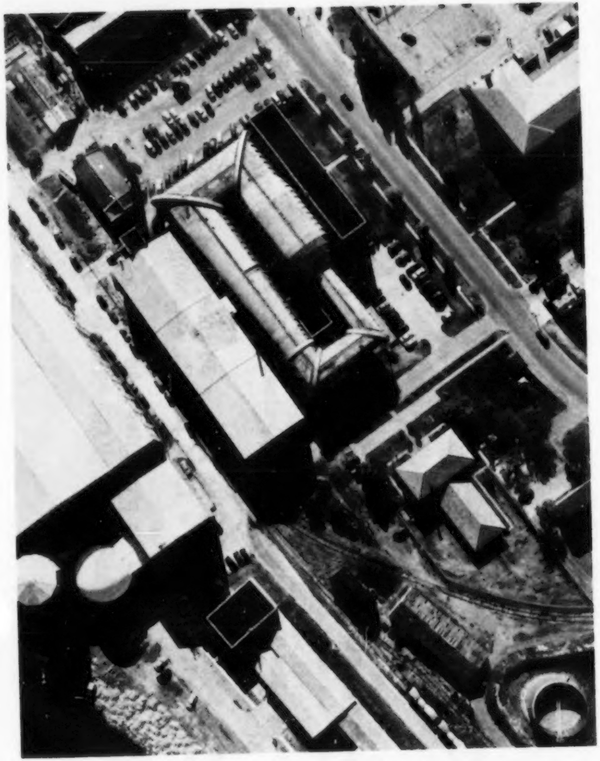

FIG. 1.—GENERAL SITE OF REPOWERED 19-FOOT PRESSURE TUNNEL

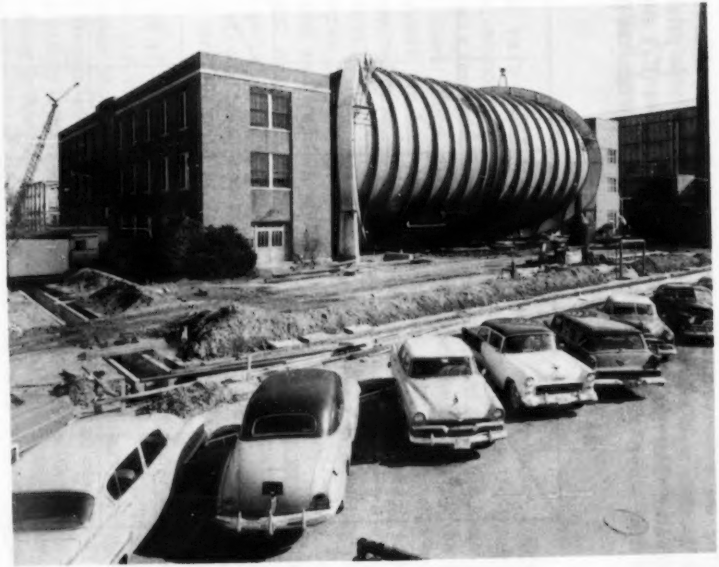

FIG. 2.—SOUTH END OF 19-FOOT PRESSURE TUNNEL

the shear and moment in the cap would depend on the relative stiffnesses of the piles and, in some cases, the cap stiffness as well. In view of the doubt concerning the relative stiffnesses of the piles, the following procedure was adopted. Design was based on the stiffness of the pipe piles (K_p) being three times greater than the stiffness of the timber piles (K_t), and then the results checked for other ranges of K_p to K_t to assure against failure for remote possibilities. The following allowable values were adopted for various ratios of K_p to K_t:

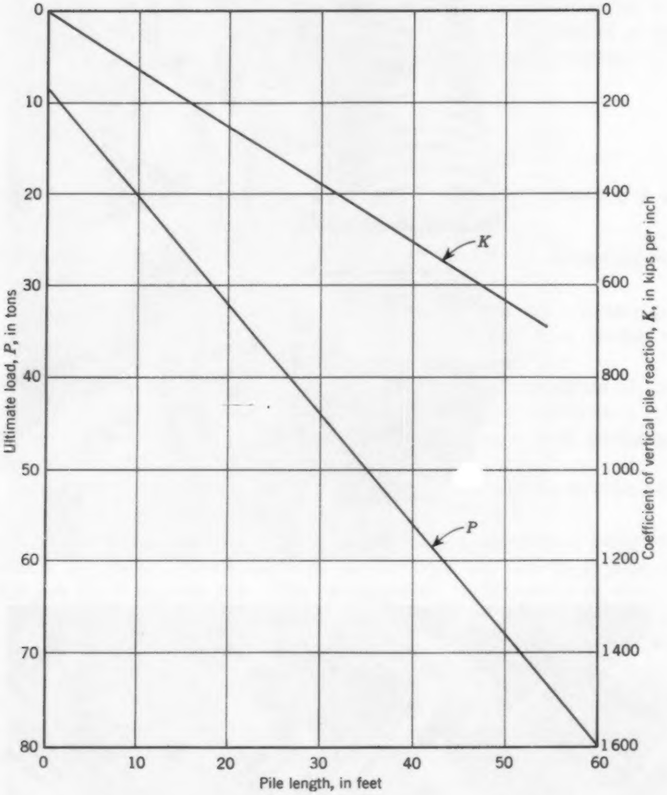

FIG. 3.—TEST DATA FROM PRIOR PROJECTS PILE CAPACITIES AND STIFFNESSES

(a) $K_p : K_t = 3$—load on pipe piles = 30 tons; load on timber piles = 20 tons; concrete fibre stress = 1,350 psi $(f'_c = 3{,}000)$; and reinforcing steel stress = 20,000 psi.

(b) $K_p : K_t = 1$—load on pipe piles = 30 tons; load on timber piles = 30 tons; and reinforcing and concrete stress as in (a). (This assured that if both type piles had similar load-deflection characteristics, the timber pile would have a safety factor of 1-1/2 against failure).

(c) $K_p : K_t = 00$—load on pipe piles = 60 tons; concrete fibre stress = 2,000 psi; and reinforcing steel stress = 26,000 psi. (This assured that should the timber piles fail, or be very weak in comparison with the pipe piles, the pipe piles were able to support the entire load without failure).

DESIGN

The design of each foundation constituted a new problem. Owing to the space limitations, standard design procedures could not be utilized; in fact, in many cases it was impossible to group the new piles near the loads because of existing utility lines. Since, also, in many cases the existing piles and caps were used to support the load while the actual underpinning was underway, ingenuity was required to space piles in a manner to permit both pile driving and cap construction.

A trial and error procedure was the general rule; piles would be spaced where possible and an investigation made. This investigation would serve as a guide to the next pile arrangement. In this manner, a gradual integrated design was developed which obtained a balance among the parameters of number of new piles, size of caps, and reinforcing required.

Two examples are included to illustrate the varied problems encountered in the design.

Problem one is of an isolated rigid footing in which incorporation of both the existing piles and cap was found feasible. This foundation supported a corner of the tunnel shown in Fig. 2. A symmetrical arrangement of piles could not be accomplished because of the presence of an electrical duct line on one side and a utility trench on the other as shown in Fig. 9. Considerable thrust loads resulting from wind loads and thermal expansion acted on the top of the pier. This necessitated that the investigation include footing moment in addition to axial loading.

The standard formula[3] for a pile supported rigid footing is:

$$P = \frac{\Sigma V}{n} \pm \frac{\Sigma M\ d}{\Sigma\ d^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which P is the total pile reaction, ΣV refers to the sum of vertical loads, ΣM denotes the sum of moments about the center of gravity of the pile group, n is the number of piles, d denotes the distance from the center of gravity to the pile, and $\Sigma\ d^2$ refers to the sum of distances squared.

This formula had to be modified to take the varying stiffnesses of the piles into account. This was accomplished by developing a similar equation in terms of deflection rather than pile load and then finding the pile load on the basis of this deflection and the particular pile's stiffness (Fig. 4). The deflection due to axial load is

$$\Delta A = \frac{P + W}{N_t\ K_t + N_p\ K_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

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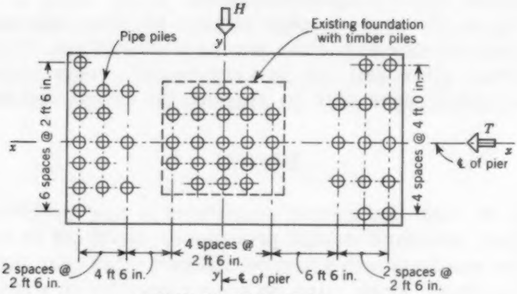
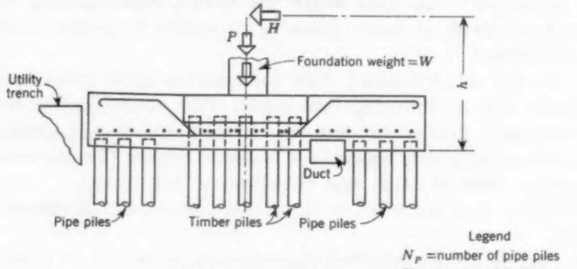
[3] Foundation Engineering, by Peck, Hanson and Thornburn, p. 333.

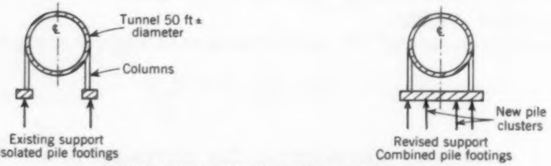

H
y
Pipe piles
Existing foundation with timber piles
x
6 spaces @ 2 ft 6 in.
4 spaces @ 4 ft 0 in.
T
x
℄ of pier
4 spaces @ 2 ft 6 in.
2 spaces @ 2 ft 6 in.
4 ft 6 in.
y ℄ of pier
6 ft 6 in.
2 spaces @ 2 ft 6 in.

(*a*) PLAN OF FOUNDATION

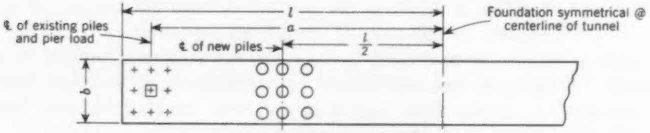

H
P
Foundation weight = W
h
Utility trench
Duct
Pipe piles
Timber piles
Pipe piles
Legend
N_P =number of pipe piles
N_T =number of timber piles

(*b*) SECTION AT CENTERLINE

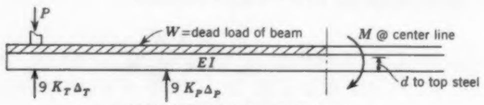
FIG. 4.—CORNER FOUNDATION (C_O) *l*


℄
Tunnel 50 ft ± diameter
Columns
Existing support
Isolated pile footings
℄
New pile clusters
Revised support
Combined pile footings

(*a*) RING ELEVATION BEFORE AND AFTER


l
a
$\frac{l}{2}$
℄ of existing piles and pier load
℄ of new piles
Foundation symmetrical @ centerline of tunnel
b

(*b*) PLAN OF FOUNDATION


P
W = dead load of beam
M @ center line
EI
d to top steel
$9\ K_T\Delta_T$
$9\ K_P\Delta_P$

(*c*) ASSUMED DESIGN CONDITION

FIG. 5.—DESIGN OF FOOTING

The deflection due to moments is

$$\Delta M = \frac{T\ h\ x}{K_t \sum_{i=0}^{N_t} x_i^2 + K_p \sum_{j=0}^{N_r} x_j^2} + \frac{H\ h\ y}{K_t \sum_{i=0}^{N_t} y_i^2 + K_p \sum_{j=0}^{N_p} y_i} \quad . . (3)$$

The solution follows readily. Deflection at any point (x, y) can be calculated from $\Delta A \pm \Delta M$, pile loading from $K\ \Delta$, and shear and moment diagrams constructed.

The complete investigation of the pile cap for the various ratios of K_p/K_t entailed considerable computations since shear, flexural and bond stresses in the footing had to be determined in addition to pile loads. All results were checked for compliance with criteria outlined as follows. (This is a synopsis of the investigation and is not intended to show the complete results).

Values of Constants.—P = 1,050,000 lb; W = 380,000 lb; T = H = 104,000 lb; h = 20 ft; d = 41 in.; A_s = 88 sq in. (6-12I50).

1. For case $\frac{K_p}{K_t} = 3$; K_p = 500 kips per in.; K_t = 167 kips per in.

Load on extreme pipe pile = 22 tons normal load O.K. < 30 tons
= 37 tons thrust acting O.K.
1/3 overload allowed
Load on extreme timber pile = 7 tons normal load O.K. < 20 tons
= 9 tons thrust acting O.K.
f_s (max) = 21,000 psi thrust acting O.K.
f_c (max) = 1,180 psi thrust acting O.K.
v (max) = 83 psi thrust acting O.K.

2. For case $\frac{K_p}{K_t} = 1$; $K_p = K_t$ = 500 kips per in. (Note that $K_p <$ 500 kips per in. is not realistic.) Moments, shears, pipe pile loads will reduce, timber pile loads will increase

Load on extreme timber pile = 15 tons normal load O.K. < 20 tons
= 19 tons thrust acting O.K.

3. For case $\frac{K_p}{K_t} = \infty$; K_p = 500 kips per in.; K_t = 0 (failure)

Load on extreme pipe pile = 26 tons normal load O.K.
= 40 tons thrust acting O.K.
1/3 overload allowed
f_s (max) = 26,000 psi
f_c (max) = 1,460 psi O.K. thrust acting
v (max) = 102 psi 1/3 overstress allowed

The second problem illustrates the solution for a combined footing utilizing elastic beam theory. This foundation supported a ring of the tunnel and an adjacent building prevented grouping of piles for an isolated footing (see Fig. 11). Piles were driven as close to the load as feasible and the footing designed as a beam to span between ring supports as shown on Fig. 5. Sum of vertical forces = 0:

$$P + W\ 1 - 9\ K_t\ \Delta_t - 9\ K_p\ \Delta_p = 0 \ . . . . (4)$$

Sum of moments @ ℄ = 0:

$$M + 9\,K_p\,\Delta_p\,\frac{l}{2} + 9\,K_t\,\Delta_t\,a - Pa - \frac{W\,l^2}{2} = 0 \tag{5}$$

By symmetry, slope @ ℄ = 0:

$$\alpha_1 + \alpha_2 + \alpha_3 = 0 \tag{6}$$

and

$$\left[\frac{\Delta_t - \Delta_p}{(a - l/2)}\right] - \left[\frac{W\,l^3}{48\,E\,I}\right] - \left[\frac{M\,(l + a)}{3\,E\,I}\right] = 0 \tag{7}$$

in which α_1 is the slope due to support flexibility with beam rigid, α_2 denotes the beam slope due to W, and α_3 refers to the beam slope due to M.

A variation of this equation development was employed. In order to obtain a solution to this indeterminate problem, the symmetry of the system was utilized to write the third equation; symmetry required that the beam slope at the tunnel centerline be zero. This equation plus two equations from statics was written in terms of pile stiffnesses and deflections (rather than loads) and solved simultaneously. The most economical pile arrangement as well as the geometrical size of the beam was again determined by trial and error. It should be noted that intuitiveness is almost a requirement in such a design. The scope of investigation is indicated by the following calculation sheet.

Values of Constants.—$E_c = 3 \times 10^6$ psi; $E_s = 30 \times 10^6$ psi; W = 5700 lb per ft; l = 30 ft; a = 27 ft 9 in.; b = 7 ft; d = 5 ft; P = 550,000 lb; $A_s = 53$ sq in.

1. For case $\frac{K_p}{K_t} = 3$; $K_p = 500$ kips per in.; $K_t = 167$ kips per in. Solving Eqs. 4, 5, and 7 simultaneously results in the following:

M = 1900 ft-kips, $\Delta_p = 0.077$ in., $\Delta_t = 0.250$ in.

Load on pipe pile = 0.077 × 500 = 38.5 kips $\approx$ 20 tons O.K. $<$ 30 tons.

Load on timber pile = 0.250 × 167 = 42 kips = 21 tons O.K. $\approx$ 20 tons.

$$f_s = \frac{M}{A_s\,j\,d} = \frac{1900 \times 12}{53 \times 0.87 \times 60} = 8300 \text{ psi O.K.} \tag{8}$$

f_c O.K. $<$ 1350 psi by inspection

Note that beam size and steel purposely oversized to furnish the required rigidity for load distribution to piles.

2. For case $\frac{K_p}{K_t} = 1$; $K_p = K_t = 500$ kips per in. (K_p less than 500 kips per in. is not realistic).—Solving Eqs. 4, 5 and 7 simultaneously results in the following:

M = 520 ft-kips, $\Delta_p = 0.048$ in., $\Delta_t = 0.110$ in.

Load on pipe pile = 0.048 × 500 = 24 kips = 12 tons O.K. $<$ 30 tons

Load on timber pile = 0.110 × 500 = 55 kips = 28 tons O.K. $<$ 30 tons

f_s and f_c O.K. by inspection. (Since M has decreased.)

3. For case $\frac{K_p}{K^t} = \infty$; $K_p = 500$; $K_t = 0$ (failure).—Problem is now solved by statics.

$$\text{Load on pipe pile} = \frac{722 \text{ kips}}{9} = 80 \text{ kips} = 40^t \text{ O.K.} < 60 \text{ tons}$$

Load on timber pile = 0

M = 6,000 ft-kips

$$f_s = \frac{M}{A_s\, j\, d} = \frac{6{,}000 \times 12}{53 \times 0.87 \times 60} = 26{,}000 \text{ psi O.K.} \approx 26{,}000 \text{ psi}$$

$f_c < 2{,}000$ psi O.K.

SPECIFICATIONS

The contractor was given the option of furnishing H-beam or concrete cast-in-place in metallic shell piling. Welded or sleeved joints between sections were permitted. Shell thickness was left to the discretion of the contractor provided it was of sufficient thickness to withstand the collapsing pressures encountered and to transmit the driving stresses; a minimum diameter of 10-3/4 in. was required for shells.

Since the piles were primarily a friction type, 85% friction and 15% end bearing, a bid length was specified; piles were to be driven to a penetration determined by load tests to failure. In addition to these load tests, the contractor was required to drive and test a 15-ft timber pile so that the design criteria could be checked.

The contractor was required to place bench marks on all structures in the vicinity so that any heaving could be determined and corrective measures taken. Bench marks were also specified to be placed integrally in all new pile caps so that settlement-time characteristics could be determined.

FIELD LOAD TESTS

Five piles were tested to failure by a test method illustrated in Fig. 6. The results of these pile tests are tabulated in Fig. 7. A careful study of these tests, in addition to the utilization of previous test results and experience, was required because of the scatter of the test data. The following conclusions were reached:

1. Contract length for piles was set at 40 ft since two 30-ft pipe piles settled in excess of 2 in. at 50 tons. A minimum safety factor of 2 was regarded as essential because of the possibility of vibratory loads. Previous tests on Langley Field have indicated an average value of approximately 1,500 psf for skin friction. Thus, it was felt that the test values obtained in excess of 2,000 psf were due to scattered pockets of cemented shell which could not be depended upon to furnish consistent support. The contractor was directed to drive all piles to 40-ft penetration regardless of number of blows per foot. It should be noted that the use of dynamic pile driving formulas had been proved unduly conservative and costly on Langley Field. Several piles driven to a resistance of less than 7 blows per ft with a Vulcan No. 1 hammer (7 tons, Engineering News Record Formula) have load tested to values in excess of 100 tons.

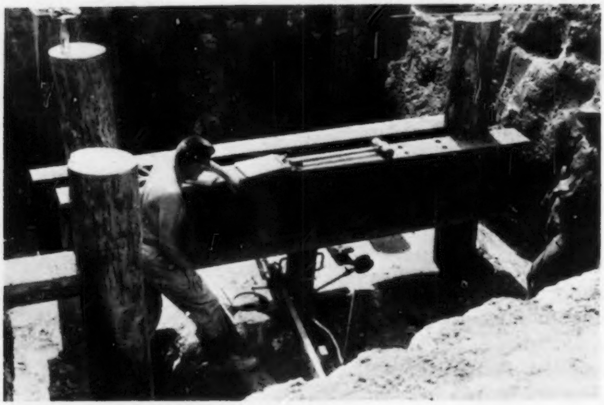
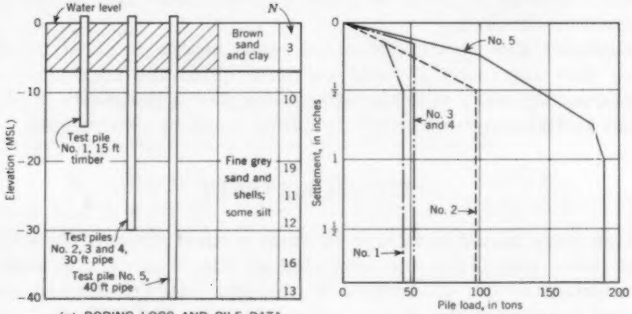


FIG. 6.—PILE TESTING. GENERAL CONSTRUCTION, 19-FOOT PRESSURE TUNNEL CONVERSION



(*a*) BORING LOGS AND PILE DATA

(*b*) PILE LOAD TESTS

Pile	Embedded Area, in sq ft	Ultimate Load-tons	Ultimate Skin Friction, in lb per sq ft	Coeff. of Vert. Pile Reaction kips per in
15 ft Timber	47	45	1920	400
30 ft Pipe	83	100	2400	500
30 ft Pipe	83	50	1200	1100
30 ft Pipe	83	50	1200	1100
40 ft Pipe	110	190	3450	1100

FIG. 7.—PILE TEST RESULTS

2. Ultimate pile capacities for piles were set conservatively at 90 tons for the pipe pile and 45 tons for the timber pile.
3. Coefficients of vertical pile reaction were taken as 1,100 kips per in. for the pipe pile and 400 kips per in. for the timber pile.

The design notes were reviewed to take account of this test data. No design revisions were required as the tests, in general, substantiated the design assumptions.

CONSTRUCTION

The contractor followed the same procedure as was utilized in the design in that he treated each foundation as a separate problem. Pipe pile was chosen because pipe was readily adaptable to driving in various lengths with small waste. The pipe piles were delivered to the site in approximately 40-ft lengths and cut to lengths as required by the headroom available at each footing.

There were approximately 600 piles driven on the project of which 200 required sectional driving. The other 400 piles supported a building addition and auxiliary equipment. The sectional piles were driven with a McKiernan-Terry 9B3 hammer to the required 40-ft penetration. Penetration was obtained without difficulty except in isolated cases. The standard piles were driven with a Vulcan No. 1 hammer. The contractor found that he could safely reduce the thickness of the pipe from 0.250 in. to 0.188 in. as the driving proceeded. A considerable saving was effected by this reduction.

Figs. 8 through 11 illustrate the field construction. Fig. 8 shows how the small boom and leads were utilized under the tunnel. These piles under the tunnel corner were driven in approximately 7-ft sections and friction sleeved between sections (a typical sleeve is visible in the background).

Fig. 9 shows the corner foundation after completion of the pile driving. The duct bank described in the design section has been uncovered and the area is ready for forming and the placing of reinforcing steel.

The incorporation of the existing pile cap into the new cap is shown in Fig. 10. The danger of voids between the caps was eliminated by drilling 2-in. diameter holes through the existing cap and forcing cement grout into the foundation.

The conditions described in the design section which necessitated a combined footing are shown in Fig. 11. The adjacent building prevented the grouping of piles under the load and, thus, a combined footing spanning between ring supports was required. Again the restricted headroom required that the piles be driven in small sections.

The involved foundation work was completed satisfactorily and with only minor field modifications. Total cost for the pile driving was approximately $150,000. It is estimated that lineal foot costs for piles in place were approximately $5.00 for the standard piles and $8.50 for the sectional piles.

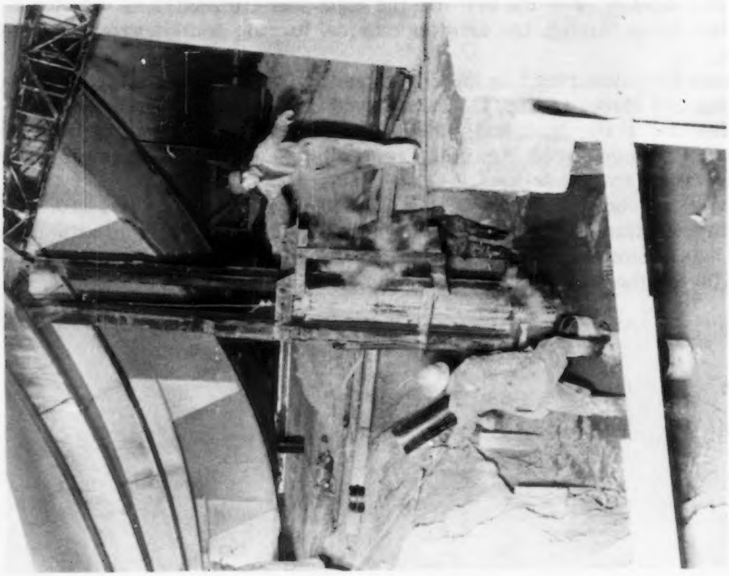


FIG. 8.—PILE DRIVING AT RINGS C AND D, 19-FOOT PRESSURE TUNNEL MODIFICATION

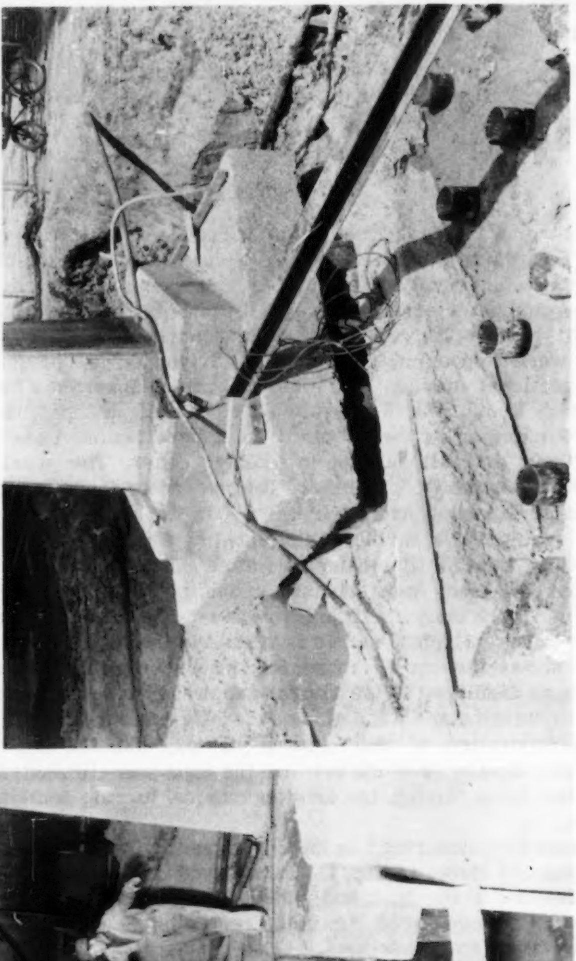


FIG. 9.—FOUNDATION—OUTSIDE CORNER RING C, 19-FOOT PRESSURE TUNNEL MODIFICATION

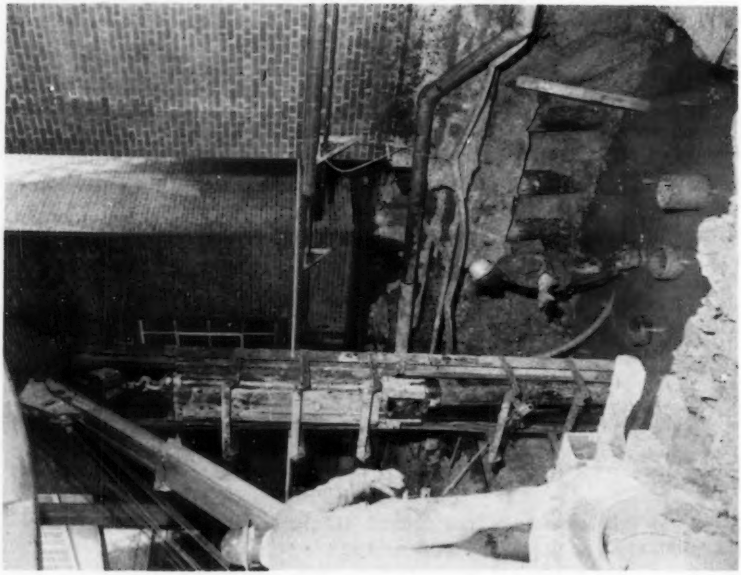


FIG. 11.—FOUNDATION—RING 59, 19-FOOT PRESSURE TUNNEL MODIFICATION

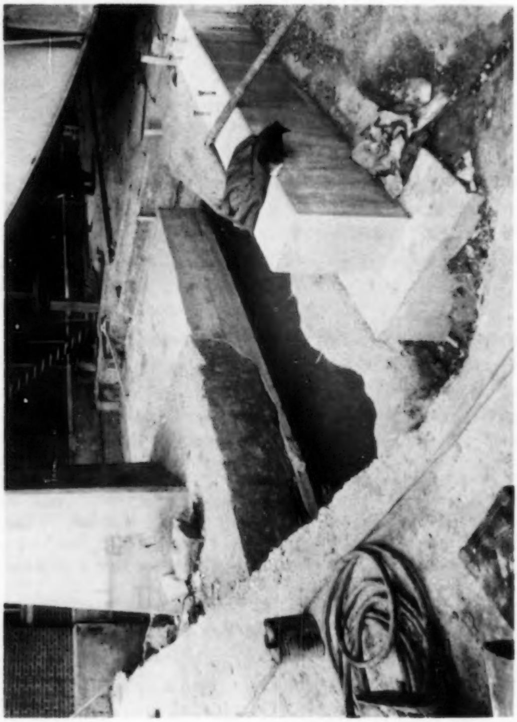


FIG. 10.—FOUNDATION—RING D-DUCT BANK AND FREON TANK FOUNDATION, 19-FOOT PRESSURE TUNNEL MODIFICATION

Settlement readings have been taken monthly on all foundations. To date, all settlements have been negligible. No settlement difficulty of any kind is anticipated.

ACKNOWLEDGMENT

The photographs included in this paper are from the files of the Langley Research Center of the National Aeronautics and Space Administration. Acknowledgement is also due the officials of the Raymond Concrete Pile Company for releasing the paper for publication. The paper was originally written for competition in the Alfred A. Raymond Award and was awarded an honorable mention.

Journal of the
SOIL MECHANICS AND FOUNDATIONS DIVISION
Proceedings of the American Society of Civil Engineers

TIMBER PILES IN PERMAFROST AT ALASKAN RADAR STATION,[a]

By Willard C. Jensen,[1] M. ASCE

SYNOPSIS

Timber piles provide stable foundation in permafrost for a large United States Air Force (USAF) "composite type" building at Kotzebue, Alaska. Pile load-deflection tests and soil temperature investigations performed during and after construction contributed valuable field data on point bearing support, pile deflections, slurry-pile adfreeze strengths, subgrade thermal changes and slurry freeze-back time by a natural or mechanical method.

INTRODUCTION

> "The key to construction techniques in the Arctic Region is cooperation with nature. The thermal conditions and subsurface relationships of an area should not be materially disturbed and if the work in the area causes or requires disturbing such relationships, provisions should be made for re-establishing them."[2]

The construction of a United States Air Force (USAF) early warning radar station near Kotzebue, Alaska (Fig. 1) began during the month of October, 1955, when timber piles were placed in a permafrost subgrade for the foundation of a "composite type" building. A continuous concrete floor slab, approximately 50,000 sq ft in area was supported by the pile footings (Fig. 2). The

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Note.—Discussion open until July 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. SM 1, February, 1961.

[a] Publication authorized by Chief of Information, Alaskan Command and Alaskan Air Command.

[1] Lt. Col., Directorate of Civ. Engrg., Headquarters USAF, Washington, D. C.

[2] "Construction Techniques," by G. W. Rathjins, THE DYNAMIC NORTH, BOOD II, June, 1956.

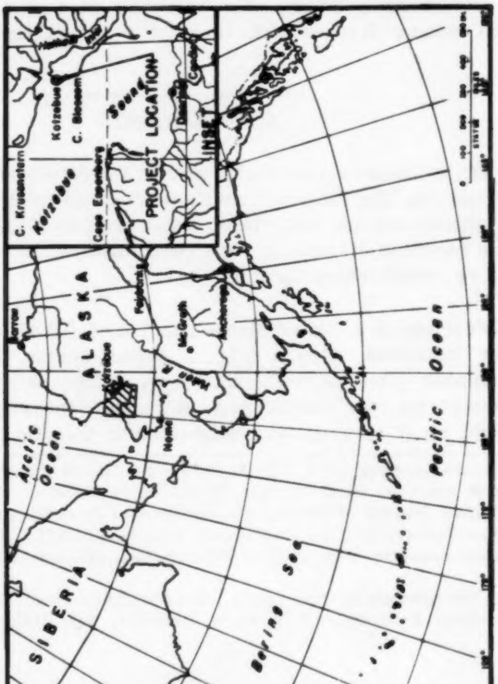

SIBERIA
ALASKA
Arctic Ocean
Bering Sea
Pacific Ocean
Barrow
Kotzebue
Nome
Fairbanks
McGrath
Anchorage
C. Krusenstern
Kotzebue
Kotzebue
C. Blossom
Cape Espenberg
Sound
PROJECT LOCATION
INSET

FIG. 1.—LOCATION MAP

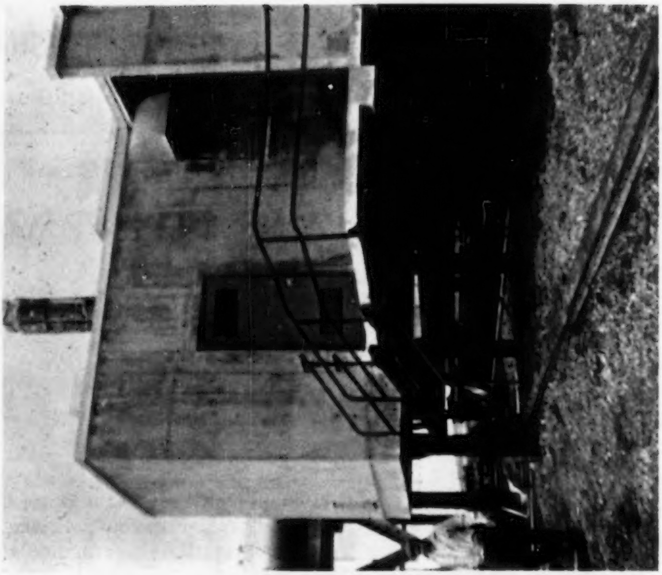


FIG. 2

5 in. floor slab was placed an average of 4 ft above the ground surface to provide an air-space beneath the building minimizing heat transfer into the permafrost. All the major station facilities were provided in the large wood framed building constructed on this floor slab. These included the operations and administration offices, a large warehouse, a power plant for both technical and domestic requirements, an automative shop, and the dormitory and dining facilities for the entire station's personnel.

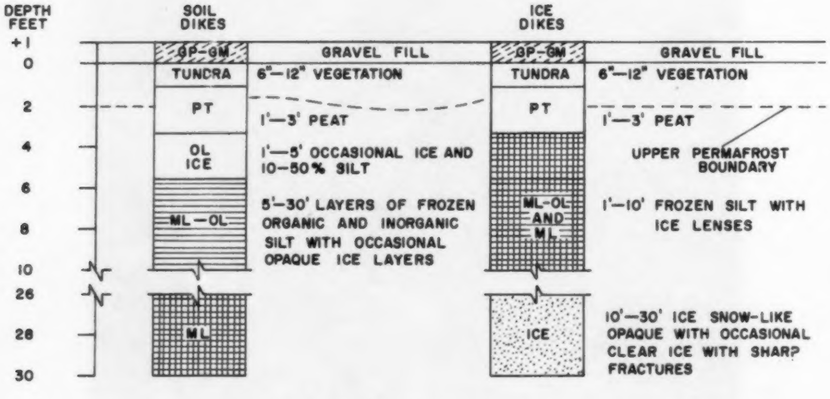
FIG. 3.—AIR VIEW OF KOTZEUBE AIR FORCE STATION

A pile testing program was included in the construction contract. Under the direction of the Corps of Engineers, United States Army, the contractor furnished materials and performed the pile load-deflection tests and soil-temperature investigations. The soil-temperature study included foundation freeze-back control and observation of long term changes in the soil's thermal regime around the piles during and after construction.

GENERAL SITE CONDITIONS

The terrain in the Kotzebue area has polygonal ground; is a treeless rolling upland dotted with numerous lakes and ponds; and exhibits poor drainage. The building site (Fig. 3) was located on a ridge (elevation 145 ft). For subgrade investigations, soil logs were taken at three test pile locations and at seven other selected pile locations in the building area. The subgrade was composed of a series of vertical silt and ice dikes running generally in a northeast direction (Fig. 4). The annual frost zone had been completely frozen for two weeks prior to the pile footing construction and remained frozen during placement of all the piling (from October 12 to December 8, 1955). Fig. 5 is a typical example of the great variations in water content and density of the vertical silt dikes.

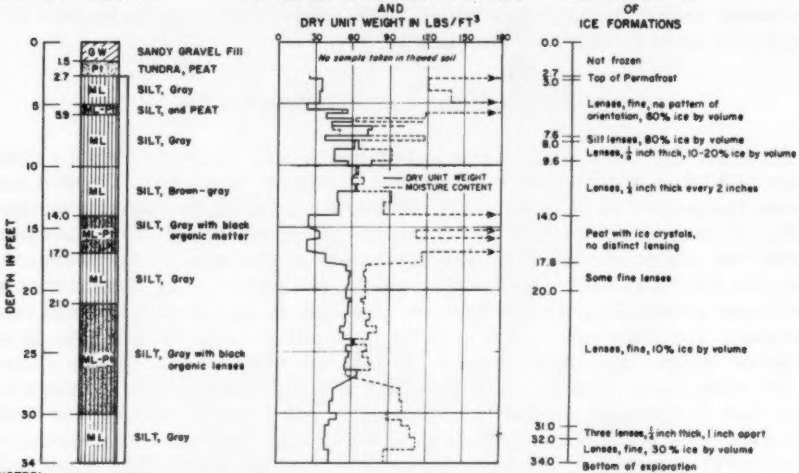
1. **LOCALE —**
   4 MILES SOUTH OF KOTZEBUE, ALASKA ON BALDWIN PENINSULA, 195 MILES NORTH-NORTHWEST FROM NOME, ALASKA.
2. **CLIMATE — TEMPERATURE —**
   MEAN ANNUAL = +20° F; EXTREMES +82 AND -50° F. MEAN FREEZING INDEX = 5823 DEGREE - DAYS. MEAN THAWING INDEX = 1950 DEGREE — DAYS.
   PRECIPITATION —
   ANNUAL AVERAGE = 7.7" WATER WHICH INCLUDES 33.9" OF SNOWFALL
3. **SOIL PROFILE —**
   COMPOSITE BUILDING IS UNDERLAIN BY A SERIES OF VERTICAL FROZEN SILT AND ICE DIKES. UPPER PERMAFROST BOUNDARY IS 1'—2' BELOW TUNDRA MAT SURFACE.


DEPTH FEET
+1
0
2
4
6
8
10
26
28
30
SOIL DIKES
GP-GM
GRAVEL FILL
TUNDRA
6"—12" VEGETATION
PT
1'—3' PEAT
OL ICE
1'—5' OCCASIONAL ICE AND 10—50% SILT
ML — OL
5'—30' LAYERS OF FROZEN ORGANIC AND INORGANIC SILT WITH OCCASIONAL OPAQUE ICE LAYERS
ML
ICE DIKES
GP-GM
GRAVEL FILL
TUNDRA
6"—12" VEGETATION
PT
1'—3' PEAT
UPPER PERMAFROST BOUNDARY
ML-OL AND ML
1'—10' FROZEN SILT WITH ICE LENSES
ICE
10'—30' ICE SNOW-LIKE OPAQUE WITH OCCASIONAL CLEAR ICE WITH SHARP FRACTURES

TYPICAL VISUAL SOIL LOGS

FIG. 4.—GENERAL SITE CONDITIONS

Climatic conditions for the period from 1943 to 1953 were obtained from a United States Weather Bureau Dept. of Commerce Station located at Kotzebue Airport, four miles north of the construction site. From late May to October, the seacoast and interior lakes are ice free and the Kotzebue Sound area has a maritime type of climate with cloudy skies or ground fog prevailing. During the long winter months, the water surrounding the peninsula is frozen, and the climate approaches the continental type. Storms having cyclonic conditions



NOTES:

1 *Acker rotary drill with 2 $\frac{1}{8}$ in. core barrels used, without water.*
2 *Drill runs limited to 1.0 foot to eliminate sample thawing.*
3 *Exploration time 20 hours, 14 and 15 September.*
4 *Unified Soil Classification used.*
5 *> Indicates moisture content above 180 %.*

FIG. 5.—SOIL DATA, TEST PILE NO. 3

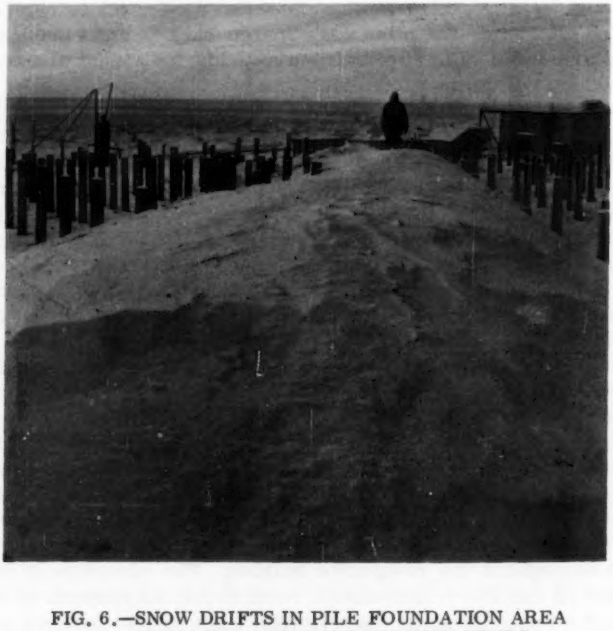


FIG. 6.—SNOW DRIFTS IN PILE FOUNDATION AREA FROM SNOW STORM WITH HIGH WINDS

result in blizzards and high winds. Fig. 6 indicates the extent drifting snow is blown around buildings or other wind obstructions. Approximately 10 ft to 15 ft of snow is shown piled in this figure.

FOUNDATION DESIGN AND CONSTRUCTION

Douglas Fir piles with an average diameter of 11 in. were placed a minimum of 30 ft in augered holes, slurried and refrozen to the permafrost's subgrade temperatures by mechanical refrigeration. An earth-augering machine (Fig. 7) was used to drill dry holes for placement of the 520 timber piles under the composite building. The holes were drilled with an earth-augering machine having an 18 in. "Alaskaug" carbide cutting auger. The contractor also furnished materials and performed the pile load-deflection tests and soil temperature investigations. After placing the piling (Fig. 8) butt down in the augered holes, the annular space around the piles was filled with slurry. 6 in poles were used to place the piles with minimum damage to moisture cell and thermocouple leads and elements. All excess creosote was wiped off each pile before placement to assure maximum adfreeze bond strengths. The slurry (Fig. 9) was consolidated as quickly as possible with a small 1-in. diameter concrete vibrator. The slurry was a gravelly sand, fully saturated with water in a concrete mixer to a consistency approximately that of a 6 in. slump concrete. Low air temperatures and gravel sizes greater than $\frac{1}{2}$ in. effected placement in annular space around piles. During last half of slurry operations, the gravelly sand was screened through a $\frac{1}{2}$ in. mesh. The gravelly sand slurry material was refrozen to ground temperatures by a refrigerated pipe-grid attached to each pile before placement. Each pile-freezing grid was connected by a portable pipe manifold to a compact refrigeration plant unit mounted on a skid. The piles were frozen-back in eight groups (Fig. 10), each group containing 40 to 70 pile-freezing grids. Individual pile freeze grids were connected to alumimum tube manifold by rubber hosing.

Seventeen Colman moisture cells and fifteen conventional cooper-constant thermocouple assemblies were used for pile slurry freeze-back control and for continuing temperature readings on selected piles after construction (Fig. 11). Each pile refrigeration group had at least one moisture cell and thermocouple assembly for temperature readings. The moisture cells were intended primarily for control of freeze-back time, but when many questionable readings were obtained in the initial pile freeze-back work, control was changed to thermocouple assemblies. The doubtful moisture cell readings were probably caused by damage to the cells or to their leads during the placing and consolidation of the slurry.

Level observations were made on selected piles to detect heave and settlement. To provide a bench mark, a capped 2 in. pipe at an elevation of 148.65 ft above mean sea level was placed 30 ft into the permafrost. The 2 in. pipe was placed in an augered hole, 16 in. in diameter and slurried to with 10 ft of the ground surface. A 4 in. pipe was then slip-cased around the 2 in. pipe and and slurried. The annular space between the two pipes was filled with crankcase oil to provide a non-heaving bench mark. Lag screws, 3 in. long and 18 in. above the ground surface were used as vertical check points on forty piles distributed throughout the composite building. The elevation of bench mark was assumed as 148.650 ft to measure vertical pile movements to the nearest thousandth of a foot. Elevation observations taken to date indicate a maximum

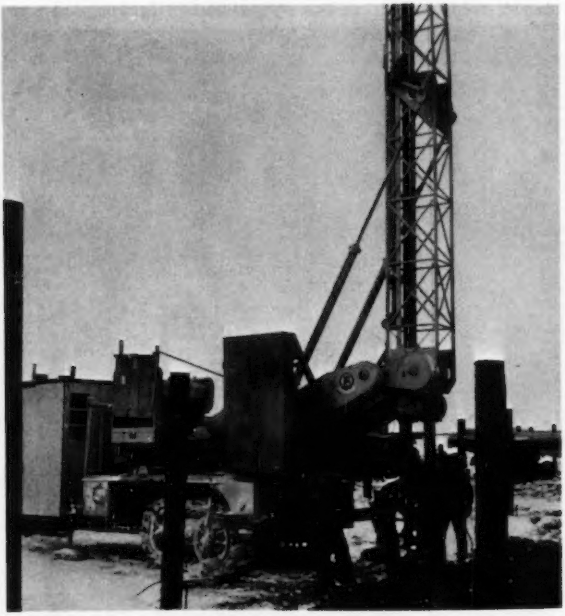


FIG. 7.—DRILLING DRY HOLES FOR PILING

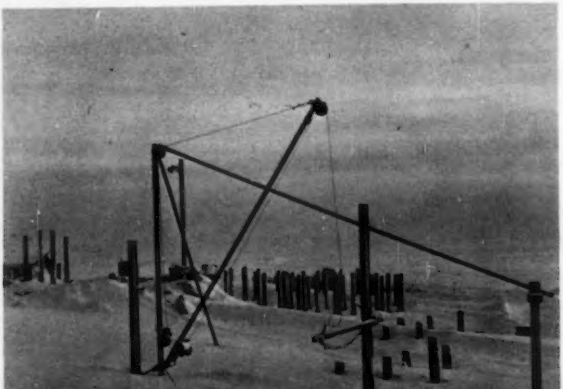


FIG. 8.—GIN-POLE FOR PLACING PILES IN AUGERED HOLES

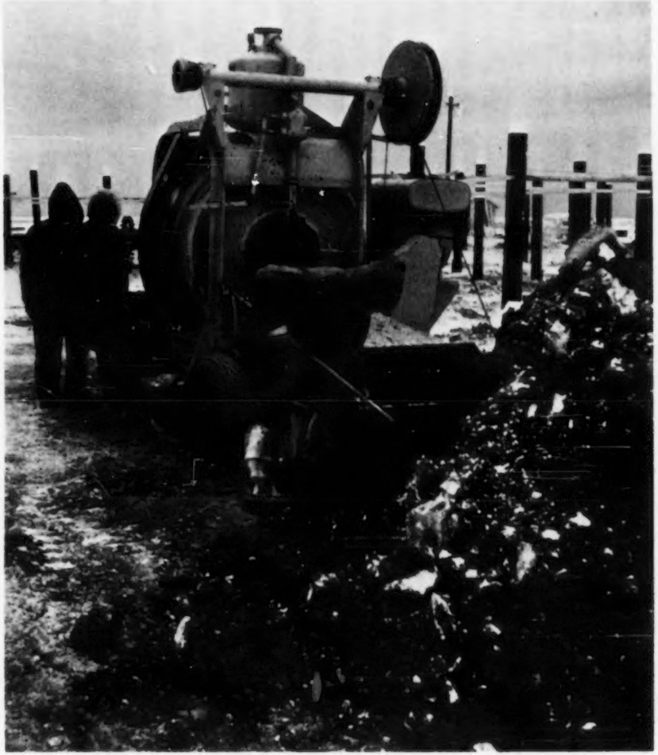


FIG. 9.—MIXING SLURRY

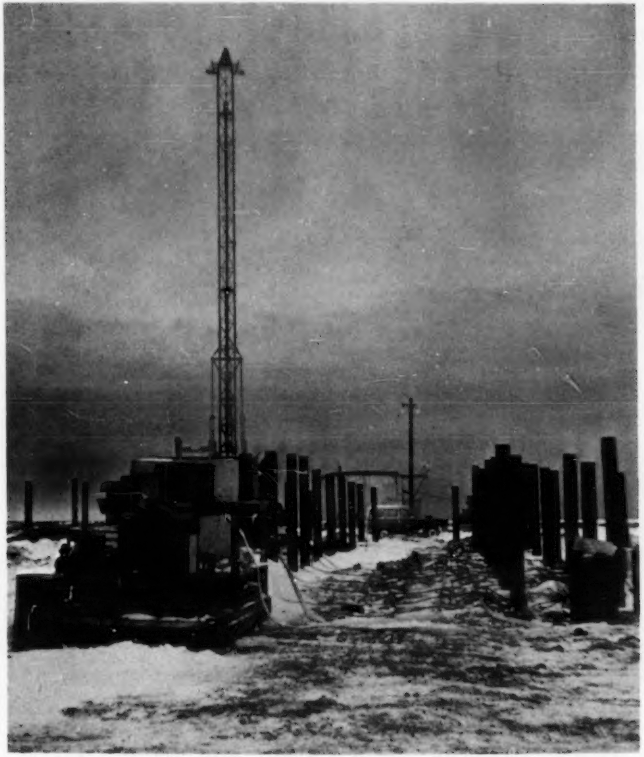


FIG. 10.—PILE FREEZE-BACK GROUP

apparent upward movement of 0.013 ft on three piles and a settlement of 0.015 ft on four piles. The actual reliable precision of elevation measurements approaches 0.01 ft, because of the sub-zero temperatures, high winds and instability of the level on thawing or frozen ground. The vertical movements of the piles were well within design tolerances because their maximum vertical movements were within the limits of precision of measurement and no appreciable heave or settlement of the piles was discernible.

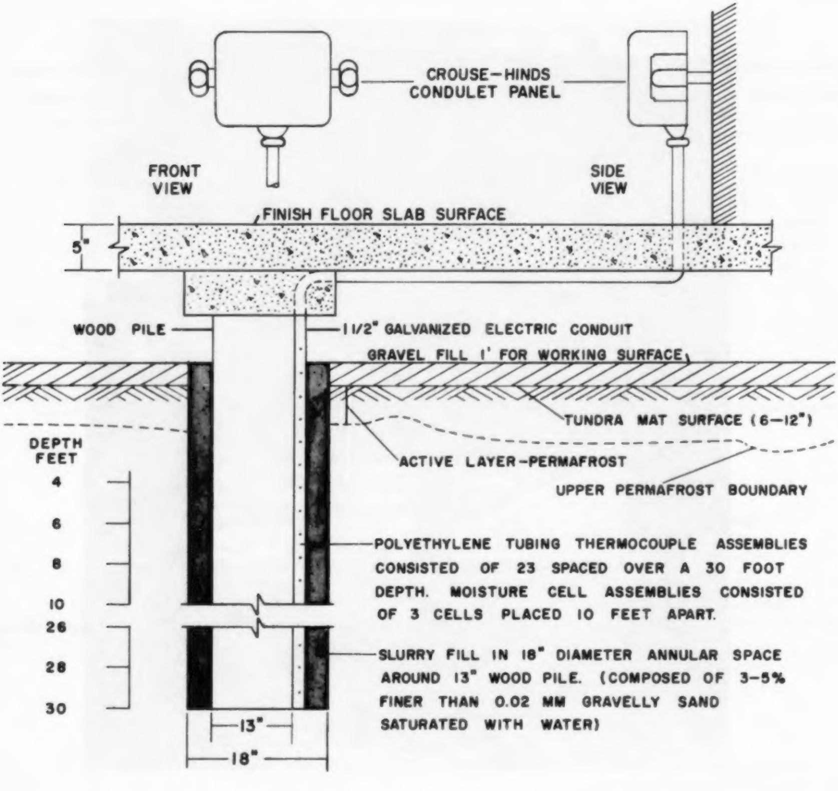

CROUSE-HINDS
CONDULET PANEL
FRONT VIEW
SIDE VIEW
FINISH FLOOR SLAB SURFACE
5"
WOOD PILE
1 1/2" GALVANIZED ELECTRIC CONDUIT
GRAVEL FILL 1' FOR WORKING SURFACE
TUNDRA MAT SURFACE (6—12")
ACTIVE LAYER-PERMAFROST
UPPER PERMAFROST BOUNDARY
DEPTH FEET
4
6
8
10
26
28
30
POLYETHYLENE TUBING THERMOCOUPLE ASSEMBLIES CONSISTED OF 23 SPACED OVER A 30 FOOT DEPTH. MOISTURE CELL ASSEMBLIES CONSISTED OF 3 CELLS PLACED 10 FEET APART.
SLURRY FILL IN 18" DIAMETER ANNULAR SPACE AROUND 13" WOOD PILE. (COMPOSED OF 3-5% FINER THAN 0.02 MM GRAVELLY SAND SATURATED WITH WATER)
13"
18"

FIG. 11.—PILE-THERMOCOUPLE-MOISTURE CELL INSTALLATION

PILE TESTING

Two point-bearing tests were made on piling confined by small amounts of unfrozen sand on the butt end. Three load-settlement tests were made with piles completely slurried and frozen-back. Three piles were selected for straightness from the contractor's stock of Douglas Fir piling. They were placed in the permafrost using the same equipment and methods as for placing the piling for the building foundation. Two of the test piles were used for the point-bearing tests without slurry, then slurried and froze-back for load-settlement tests. The third test pile was the only one given the load-settlement test for 20, 60, and 100 ton loadings (Fig. 12). Test pile loads (maximum 100 tons) were obtained by loading box with gravel. Hein-Werner hydraulic jack Model 100:2AA was used to transmit load from box to the test pile. Table 1 is a summary of the pile loading tests.

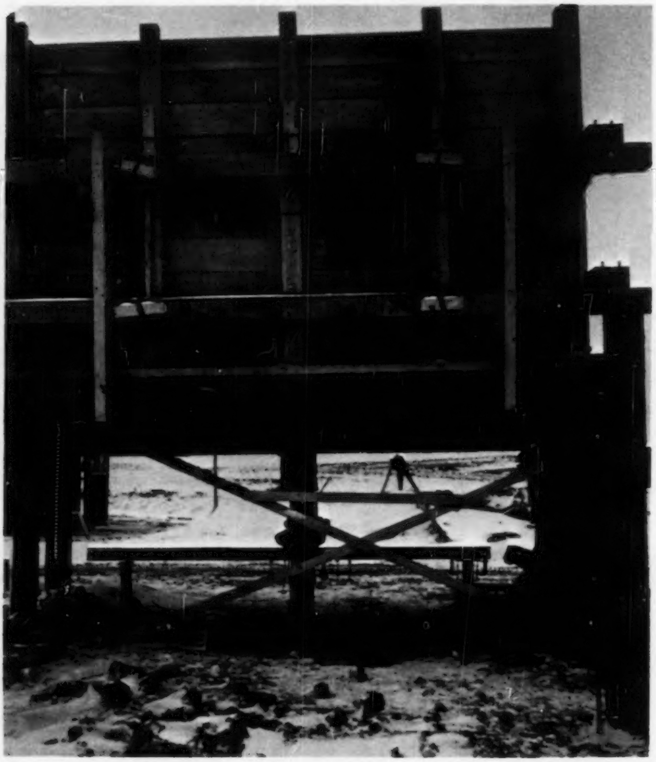


FIG. 12.—APPLYING TEST PILE LOADS

The piles had excessive deflections at maximum loads when tested in point bearing. The settlements were substantially more than could be accepted in practical construction. Initially, the excessive deflections were considered to be the combined result of the compression and displacement of the sand cushion, compression of permafrost cuttings left in hole by the auger, the displacement of undisturbed permafrost below the pile-tip or a combination of these two. Laboratory tests indicated the deflections of the field point-bearing test were several times that which might be attributed to compression and displacement of the sand cushion. Further, the average pile-tip contact pressure ranged from 242 psi to 389 psi at maximum test loads. Tests have shown that ice will deform plastically at stress levels far below these values. The excessive deflections probably resulted from either the consolidation of loose cuttings from the auger or plastic deformation of the area around the pile tip.

TABLE 1.—SUMMARY OF PILE LOAD—DEFLECTION TESTS

1. General: Loads were applied in 5 ton increments at 5 min intervals up to maximum loadings; then reduced to zero by 5 ton steps every 5 min.

2. Test Pile Data:

Pile	Total Length, in feet	Inbedded Length, in feet	Butt Diam, in inches	Mid Diam, in inches	Tip Diam, in inches	Plate Diam, in inches	Thickness of Sand Layer Under Pile, in inches
No. 1	34.5	31.0	12.25	11.0	9.7	12.5	2
2	34.9	31.5	13.3	12.1	10.9	14.5	9
3	35±	31.4	13.0	11.9	10.6		0

3. Point Bearing Test:

Pile	Maximum Load, in tons	Plate Pressure, in psi	Gross Settlement, in inches	Net Settlement, in inches	Remarks
No. 1	20	326	0.019	0.919	Max load held 5 min
	60	489	5.00	4.700	Max load held 7 hr
No. 2	20	242	6.3		Max load held 20 hr No measurements taken during unloading.

4. Load Settlement Test:

Pile	Maximum Load, in tons	Average Adfreeze Strength, in psi	Gross Settlement, in inches	Net Settlement, in inches	Remarks
No. 1	20	3.1	0.009	0.006	Max load held 5 minutes Load was reduced to 1/2 ton not zero.
	60	9.4	0.108	0.035	Max load held 86.5 hours.
No. 2	60	8.3	0.087	0.037	Max load held 50 hours.
No. 3[a]	20	3.2	0.016	0.002	Max load held 5 minutes.
	60	9.4	0.091	0.024	Max load held 24.5 hours.
	100	15.9	0.199	0.058	Max load held 72.6 hours.

[a] Effective length of pile in frozen ground was 31.4 - 3 = 28.4 ft because thaw depth just before tests was 3 ft.

Unless the bottom of the hole has been properly prepared by a special cleanout auger, placement of a small amount of water, or very wet slurry: point resistance should not be used as additive to the bearing value of production piles. Because insufficient knowledge was available concerning point support of piles in permafrost, and in view of the ease that added bearing capacity was obtained in soil-pile bond, as by slight increase in length of pile, the pile end-bearing contribution was not considered.

The maximum gross settlements cumulative from all cycles in the load-settlement tests did not appreciably exceed 0.1 in. (0.108) with the 60 ton loads and reached only 0.225 in. (0.002 + 0.024 + 0.199) with the 100 ton loads. The magnitude of these deflections were well within design tolerances. Because the deflections under the maximum 60 ton and 100 ton loads were approaching negligible settlement versus time rates when the test loads were removed, the slurry material with the 9 psi to 16 psi average adfreeze bond strength was considered capable of supporting three times the 20 ton to 30 ton pile design loads.

An over-design in foundation support was expected because the piles were purposely placed thirty feet in the ground to assure a stable foundation even in the event progressive thermal degradation occurred into the extensive ice dikes at the 15 ft depth. Temperature readings taken since the 1955 load-settlement tests indicate a general increase of the seasonal thaw depth to 5 ft with insignificant changes in the lower permafrost thermal regime. Three thermocouple stations had readings that lowered the permafrost table 7 ft, 8 ft and 9 ft below the tundra mat. The combined effect of the unusually warm winters in 1958 and 1959, and heat absorption from the composite building probably increased the annual frost zone from the 2 ft to 5 ft depth. The 1959 temperature records appear to indicate that the permafrost table is stabilized at 5 ft. However, an effective pile length of 25 ft in the frozen ground still provides an entirely adequate support for the design loads.

CONCLUSIONS

Placing the piles 30 ft into the ground to assure support if thermal degradation penetrated to a depth of 15 ft no longer appears to be a mandatory requirement in areas in which the mean annual temperatures are 22F or lower. Temperature readings recorded in 1959 have not indicated any significant thermal increase in the permafrost at the 15 ft depth at which the melting of extensive ice dikes could cause serious damage to the foundation. For a more economic foundation construction in the Kotzebue area, future designs of pile lengths can be based primarily on the tangential adfreezing strength and embedment of piles into the permafrost at least four times the active zone depth.

One pile control group placed and slurried during the period of November 19, and December 8, 1955 was completely frozen without mechanical refrigeration in approximately 100 hr. Mechanical refrigeration generally produced complete freeze-back within 48 hr. As a result of the data obtained from this project, empirical surveys indicating the required time to freeze slurries without artificial refrigeration were developed by R. F. Scott.[3]

Present experimental date and criteria for estimating the point bearing capacity of piles in permafrost are inadequate. Until more data is available concerning pile-tip contact pressures for permafrost, ice deforming plastically, or both, point bearing should not be considered as an appreciable additive to the bearing value of production piles.

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[3] "Freezing of Slurry Around Wood and Concrete Piles," by Ronald F. Scott, Misc. Paper No.13, Arctic Constr. and Frost Effects Lab., New England Div., Corps of Engrs., U. S. Army, May, 1956.

The slurry material in the 100 ton load test had an average tangential adfreezing strength greater than 16 psi if the assumption is accepted that the pile settlement was essentially complete when the load was removed. The soil-pile bond of the slurry material[4] (gravelly sand) is approximately 35 psi. The relation of the slurry's bond to the surrounding permafrost regime is unknown. Because sand has greater adfreezing strength[4], temporary compressive, tensile and shear resistance[5] than silt and ground ice, pile failure should result primarily from plastic floe, shear or both effects from the natural ground ice and silt permafrost surrounding the slurry material. Perhaps, in permafrost of low adfreeze strength, by enlarging the diameter of the augered hole and filling the pile's annular space with select materials, the soil-pile bond can be appreciably increased whereas at the same time the slurry's shear and bond stress on the permafrost subgrade is decreased.

The average skin friction values for timber piles that have been published in various technical literature vary from 0.25 ton per sq ft (heavy clays and compact sands), to 0.15 ton per sq ft (loose sands to sandy clay) and to an insignificant value for silty soils.[6] As a comparison, it is interesting to note that the 16 psi (1.1 ton per sq ft) average adfreeze bond strength computed from the 100 ton load test is four to seven times greater than the average friction pile resistance of similar but unfrozen soil types.

With an increasing number of military construction projects being constructed in permafrost areas and with added construction resulting from the steady growth of the state of Alaska and northern Canadian industries, further field and laboratory tests and data will probably eliminate many of the unknowns that have become apparent from the permafrost and ice tests made to date (1960).

ACKNOWLEDGMENTS

The writer wishes to acknowledge the use of material from the U. S. Army Engineer District, Alaska, Corps of Engineers, in the preparation of this paper. Particular acknowledgment is given to Mr. John C. Ireton, Mr. Donald B. Slawson, and Mr. Erwin L. Long, Alaska District, for discussing and making available data concerning the pile testing program for Kotzebue Air Force Station and for their assistance in the numerous technical problems associated with the design and construction of foundations on permafrost.

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[4] "Review of Certain Properties and Problems of Frozen Ground, including Permafrost," by Purdue Univ. Engrg. Experiment Station, Sipre Report 9, Snow, Ice, and Permafrost Research Establishment, Corps of Engrs., U. S. Army, March, 1953, p. 94.

[5] "Investigation of Description, Classification, Strength Properties of Frozen Soils," Report of Investigations Fiscal Year 1951, Arctic Constr. and Frost Effects Lab., New England Div., Corps of Engineers, Sipre Report 8, by Snow, Ice and Permafrost Research Establishment, Corps of Engrs., U. S. Army, March, 1953, pp. 36, 43, 46.

[6] "Soil Mechanics," by D. F. Kryninc, McGraw-Hill Publishing Co., Inc., 1947, p. 221.

Journal of the
SOIL MECHANICS AND FOUNDATIONS DIVISION
Proceedings of the American Society of Civil Engineers

NEW METHOD OF CONSOLIDATION—COEFFICIENT EVALUATION

By Ronald F. Scott,[1] M. ASCE

SYNOPSIS

At the present time (1961) there are two usual methods of determining the coefficient of consolidation of a clay soil from a laboratory consolidation or oedometer test. These are the logarithm of time (A. Casagrande, F. ASCE) and square root of time (D. W. Taylor) methods, which depend on the observer's obtaining readings of the compression dial as a function of time for 24 hr and roughly 1 hr, respectively, depending on the variety of the test used.

Following a method suggested by J. C. Jaeger in connection with the unsteady state drawdown of a well, a new technique has been devised for evaluating the consolidation coefficient from any type of consolidation test for which a solution of the K. Terzaghi, Hon. M. ASCE, consolidation equation for the region under the given boundary and initial conditions is available. This method utilizes the ratio of compressions taking place up to different times, so that continuous compression-time readings are not required. It would appear to have value as a general technique of obtaining the consolidation coefficient, besides providing spot-checks on the methods used at the present time.

INTRODUCTION

Two methods have been used for a number of years for the purpose of evaluating coefficients of consolidation from the laboratory consolidation or oedometer test. They are the logarithm of time (due to Casagrande) and the square root of time (due to Taylor) methods, both of which require the reading of compression dials at relalatively frequent intervals of time during the pro-

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Note.—Discussion open until July 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. SM 1, February, 1961.

[1] Asst. Prof. of Civ. Engrg., California Inst. of Technology, Pasadena, Calif.

cess of consolidation of a clay sample. In the first method the determination of the coefficient of consolidation normally requires that compression readings be carried through to the end of the 24-hr increment of load in order that the slope of the compression curve attributed to secondary compression of the soil be accurately evaluated on a curve of compression versus logarithm of time. In the second of the two methods, readings of a compression dial are taken up to a time corresponding to at least 90% of primary compression of the test sample under the increment of loading being considered. The 90% consolidation time varies, of course, with different samples and with different thicknesses of sample, but frequently the compression dial is read for a period of up to 1 hr after the initiation of loading. In this method it is sometimes difficult, in the case of rapidly consolidating samples, to obtain adequate readings in the early part of compression because of the speed of rotation of the compression-dial pointer.

With both methods the readings of the compression dial and time are used in connection with the basic consolidation theory to establish a real time corresponding to the 50% and 90% primary compression points, respectively, from which the coefficient of consolidation of the soil may be obtained through the time factor relationship.

Both methods possess some disadvantages. Should a few readings of the compression dial, particularly in the early stages of compression, be missed, frequently the determination of coefficient of consolidation for that particular load increment may not be carried out. Occasionally, when the square-root-of-time method is being used, a sample is encountered in which the straight-line portion of the compression curve is hard to distinguish. If this is the case, it becomes difficult or impossible to estimate with certainty a value of the coefficient of consolidation for the soil without continuing the readings for the purpose of obtaining a logarithm-of-time plot. The logarithm of time versus compression dial method suffers from the disadvantage, particularly in commercial laboratories, of requiring dial readings over a 24-hr period although this difficulty is being increasingly avoided with the growing use of automatic recording equipment.

A new method to be described herein can be used in conjunction with the foregoing methods to give rapid "spot checks" of computations of coefficient of consolidation, or, by itself, enables such coefficients to be computed readily at different points in the process of compression of the sample. Its advantages and disadvantages become apparent as the discussion proceeds.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, in the Appendix.

DERIVATION OF NEW METHOD

By the use of certain simplifying assumptions about the behavior of fine-grained cohesive soils under one-dimensional compression Terzaghi obtained the one-dimensional consolidation equation[2]

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

which is analogous to the general Fickian diffusion equation formulated to describe any diffusion process. In Eq. 1, u is the excess over hydrostatic pore water pressure, t denotes time, c_v is the coefficient of consolidation for drainage in the z-direction, and z represents distance along the z-axis. A solution to Eq. 1 depends on the initial distribution of the diffusing material or property throughout the mass of the medium in which diffusion is taking place, and on the conditions regulating the flux of the diffusing material at the boundaries of the medium.

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[2] "Erdbaumechanik," by K. Terzaghi, F. Deuticke, Vienna, 1925.

Solutions applicable to consolidation problems have been obtained for Eq. 1 under various initial distribution and boundary conditions by O. K. Fröhlich.[3] In the conventional consolidation test, carried out on a sample from which double drainage is permitted, the initial distribution and boundary conditions are:

For $0 < z < 2\ H,\ t < 0$

$$u_{z,t} = \Delta p \quad \text{(2a)}$$

For $z = 0,\ t > 0$

$$u_{0,t} = 0 \quad \text{(2b)}$$

For $z = 2\ H,\ t > 0$

$$u_{2\ H,t} = 0 \quad \text{(2c)}$$

in which z is distance along the axis of flow, measured from the surface of the sample, 2 H denotes the thickness of the sample, $u_{z,t}$ is the excess over hydrostatic pore pressure at depth z, and Δp represents the initially applied load increment.

Under these conditions, the solution to Eq. 1 gives $u_{z,t}$. However, in the consolidation test, interest more frequently centers on the space-average value of $u_{z,t}$ throughout the depth of the sample, $\bar{u}_t$. The ratio of $\bar{u}_t$ to the space-average of the initial excess over hydrostatic pore water pressure (in this case Δp) indicates the state of excess over hydrostatic pore pressure, as consolidation proceeds. Thus, the following expression can be written:

$$\text{Degree of excess over hydrostatic pore pressure} = \frac{\bar{u}_t}{\Delta p} \quad \text{(3)}$$

As the excess over hydrostatic pore pressure dissipates the sample compresses or consolidates, and the space-average of the degree of consolidation or consolidation ratio, U, can be obtained from Eq. 3 as follows:

$$U = 1 - \frac{\bar{u}_t}{\Delta p} \quad \text{(4)}$$

Using the initial and boundary conditions, Eqs. 2, 3, and 4, the solution to Eq. 1 can be written in terms of degree of consolidation:

$$U = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2}\ e^{-M^2 T} \quad \text{(5)}$$

in which m is an integer of summation, M denotes a summation term equal to $\frac{\pi}{2}$ (2 m + 1), and T is a time factor described by

$$T = \frac{c_v\ t}{H^2} \quad \text{(6)}$$

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[3] "Theorie der Setzung von Tonschichten," by K. Terzaghi and O. K. Fröhlich, F. Deuticke, Vienna, 1936.

Since U is a function of T only in Eq. 5, if it is assumed that c_v and H remain constant, Eq. 5 may be written in the abbreviated form

$$U(T) = f(T) \qquad (7)$$

In a consolidation test, a dial gauge is usually used to measure the compression taking place in the sample as it consolidates. If the initial dial reading is d_s at time $t = 0$, the dial reading taken to be d_p at the end of primary compression (corresponding to a degree of primary consolidation equal to unity) and the dial reading at some intermediate time t is d_t, then $(d_s - d_t)$ is the compression which has taken place at time t.

At this time, the degree of consolidation can be expressed as

$$U(T) = \frac{d_s - d_t}{d_s - d_p} \qquad (8)$$

in which T is the time factor corresponding to t.

Jaeger has studied the problem of determining the "storage" and "transmission" coefficients for an aquifer penetrated by a well (a "line source").[4] Jaeger proposes a manipulation of the well equations to obtain the coefficients by considering the ratio of the drawdown distances of the water table at some distance from the well at two times, say t, and N t, after the initiation of drawdown. The problem is essentially similar to that of obtaining the coefficient of consolidation of a cohesive soil when the solution to the relevant diffusion equation is known under the given boundary conditions. As a result, the method described by Jaeger can be adapted to consolidation diffusion processes.

While this paper outlines the application of the method to the problem of obtaining the coefficient of consolidation from the oedometer test, it is emphasized that the method is quite general, and can be used in any diffusion problem in soil for which the boundary conditions are known and a solution has been obtained. A further example of this is given subsequently.

If, in a consolidation test on a soil sample which is behaving according to Terzaghi's consolidation process, the amount of primary compression at a time t is divided by the amount occurring at a time N t, in which N is a number > 1 (not necessarily an integer), the quotient can be related to Eq. 7 as a result of Eq. 8:

$$\frac{U(T)}{U(N\,T)} = \frac{f(T)}{f(N\,T)} = \frac{d_s - d_t}{d_s - d_{N\,t}} \qquad (9)$$

From Eq. 5, we can evaluate the left-hand side of Eq. 9 for convenient values of N (usually, but not necessarily integers) such as 2, 3, 4, etc. These ratios can be plotted versus time factor T in the usual way (Fig. 1) giving a family of curves which are described by the N-values written adjacent to them.

Each computed value of the ratio $U(T)/U(N\,T)$ is plotted opposite the time scale at T. It follows that, for $N = \infty$, $U(N\,T) = 1$ and

$$\frac{U(T)}{U(N\,T)} = U(T) \qquad (10)$$

so that a plot of Eq. 5 is given for $N = \infty$. This curve, the usual consolidation versus time curve, is also of value and its use is indicated herein.

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[4] "The Analysis of Aquifer Test Data on Thermal Conductivity Measurements which use a Line Source," by J. C. Jaeger, Journal of Geophysical Research Vol. 64, pp. 561-564.

In order to determine the coefficient of consolidation for a particular soil, we determine the initial dial reading d_s at a time $t = 0$ and the dial readings, d_t and d_{Nt}, at two subsequent times, the latter of which is the number N times the former. The ratio herein termed the "compression ratio" is then computed from the right-hand side of Eq. 9 using these observed values. The curves in Fig. 1 are then entered with the appropriate value of compression ratio and the value of N which was chosen in order to locate a point on the graph corresponding to a definite value of T. Substitution of the appropriate t and H^2 for the sample in Eq. 7 will then enable the value of coefficient of consolidation to be computed. This determination can be made at several different times and for different values of N, yielding a range of values of coefficient of consolidation which can either be corrected in the manner described subsequently, or may, for rough determinations, be averaged to give an approximate evaluation of coefficient of consolidation.

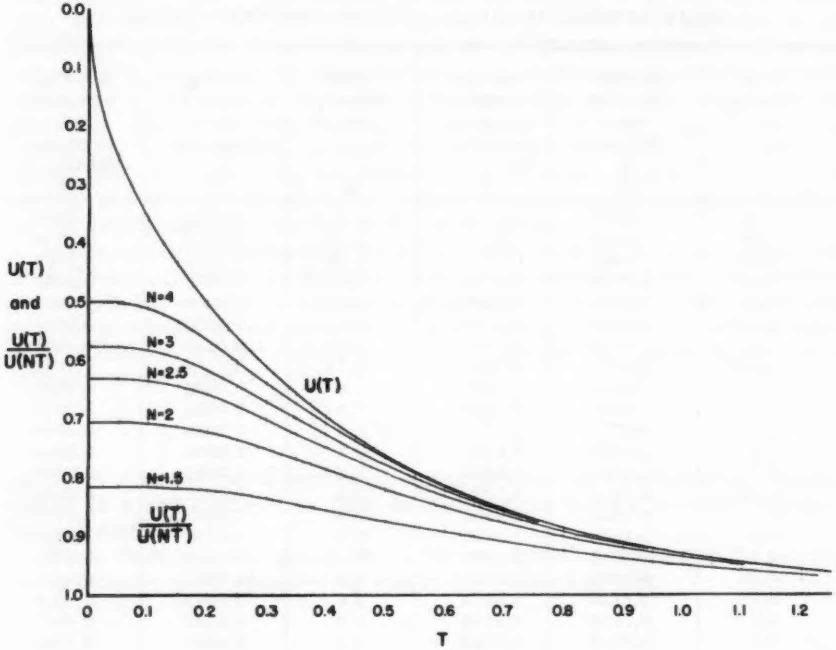

FIG. 1.—DEGREE OF CONSOLIDATION AND CONSOLIDATION RATIO VERSUS TIME FACTOR.

For convenience in carrying out computations of the amount of primary compression which has taken place, the plot of Eq. 5 in Fig. 1 may be used. When a coefficient-of-consolidation ratio has been determined according to the method described previously, the extension of the same time-factor ordinate will give an intersection on the U(T) curve corresponding to the amount of consolidation which has taken place according to Eq. 1. For the convenience of users of Fig. 1 who may wish to determine consolidation ratios at other N-values, Table 1 gives the consolidation-time factor relation to four places of decimals.

Any determination of the coefficient of consolidation implies that the dial reading d_s (usually called the "corrected zero point") at the beginning of primary consolidation is known.

Both standard methods of curve-fitting involve a determination of this dial reading based on the near-parabolic form of the U versus T curve at small values of time factor. That technique may also be used with the present method as follows: If any two compression readings are made at times corresponding to a value of N of 4 above, equal amounts of compression take place in the time intervals provided that both readings are taken in the period up to a time factor of about 0.2. The compression occurring between the two times is subtracted from the dial reading at the first time to give the corrected zero dial reading. Other values of N can, of course, be used with other compressions to allow the initial dial reading to be computed, if the parabolic nature of the curve is used.

TABLE 1.—SOLUTION OF EQ. 5 UNDER CONDITIONS OF EQS. 2

Time Factor T (1)	Average Excess Pore Pressure, $\bar{u}$ (2)	Average Degree of Consolidation, U (3)	Time Factor, T (1)	Average Excess Pore Pressure, $\bar{u}$ (2)	Average Degree of Consolidation, U (3)
0.01	0.8872	0.1128	1.2	0.0420	0.9580
0.02	0.8405	0.1595	1.25	0.0371	0.9629
0.03	0.8046	0.1954	1.35	0.0290	0.9710
0.04	0.7744	0.2256	1.4	0.0256	0.9744
0.05	0.7478	0.2522	1.5	0.0200	0.9800
0.075	0.6910	0.3090	1.6	0.0156	0.9844
0.1	0.6430	0.3570	1.65	0.0138	0.9862
0.125	0.6011	0.3989	1.75	0.0108	0.9892
0.15	0.5631	0.4369	1.8	0.0096	0.9905
0.2	0.4962	0.5038	2.0	0.0058	0.9942
0.25	0.4378	0.5622	2.1	0.0046	0.9954
0.3	0.3869	0.6131	2.2	0.0036	0.9964
0.4	0.3021	0.6979	2.25	0.0032	0.9969
0.45	0.2671	0.7329	2.4	0.0022	0.9978
0.5	0.2361	0.7640	2.5	0.0017	0.9983
0.6	0.1845	0.8155	2.7	0.0010	0.9990
0.7	0.1441	0.8559	2.75	0.0009	0.9991
0.75	0.1274	0.8726	2.8	0.0008	0.9992
0.8	0.1126	0.8874	3.0	0.0005	0.9995
0.9	0.0880	0.9120	3.2	0.0003	0.9997
1.0	0.0688	0.9313	3.3	0.0002	0.9998
1.05	0.0608	0.9392	3.6	0.0001	0.9999
1.1	0.0537	0.9463			

However, implicit in the present method of computing the coefficient of consolidation is the criterion that an initial dial reading must be chosen or computed such that consistent equal values of the coefficient of consolidation result from a comparison of compressions at different values of N. Thus, if the computation of the value of the coefficient of consolidation yields different values at different times during the consolidation period, then most probably, if no other errors have been made, the value of initial dial reading is in error and must be corrected. It will be found that an initial assumption of the zero dial reading which is too high will give coefficients of consolidation at small times which are too large and larger than those computed for subsequent times, since at later times the effect of a small change in the initial dial reading is less. If this appears in the computations, the value of the initial dial reading must be lowered. Conversely, if an initially too-low zero dial reading is taken, then the coefficients of consolidation computed for small times will be smaller than those obtained for larger times. It is generally not difficult to obtain a satisfactory initial dial reading. If we attempt to compute the function $U(T)/U(N\ T)$ in the region in which $U(T)$ approximates a parabola, we find of course that it equals $\sqrt{1/N}$. In other words, if the two times which are chosen for the computation of coefficient of consolidation for any value of N are such that the larger time falls within the approximately parabolic region of consolidation, the function curve for that value of N is essentially a horizontally straight line at the value of $\sqrt{1/N}$ and no determination can be made of the coefficient of consolidation; this is shown clearly in Fig. 1. However, it will be found that this places only a small restriction on the determination of coefficient of consolidation.

It was mentioned previously that the method outlined is generally applicable to problems for which the diffusion equation, boundary equations, and solution are known. For example, if a modification is made to the triaxial test apparatus so that a sample of cohesive soil drains radially to the outer surface while it consolidates, but not vertically, then the diffusion Eq. 1 must be written in cylindrical coordinates

$$\frac{\partial u}{\partial t} = c_r \left[\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right] \ldots\ldots\ldots\ldots\ldots \quad (11)$$

in which r is the radial coordinate and c_r denotes the coefficient of consolidation in a radial direction (that is, based on the horizontal permeability of the sample).

If the radius of the sample is a, then the initial conditions in the sample on application of the chamber, or confining pressure Δp are

For $r < a$, $t < 0$

$$u_{r,t} = \Delta p \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12a)$$

For $r = a$, $t > 0$

$$u_{a,t} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12b)$$

With these boundary conditions and utilizing expressions similar to Eqs. 3 and 4, the solution (derived by H. S. Carslaw and Jaeger[5]) of Eq. 11 for the average degree of consolidation $U(T_r)$ throughout the sample at time t is

$$U(T_r) = 1 - 4 \sum_{n=1}^{\infty} \frac{1}{\beta_n^2} e^{-\beta_n^2 T_r} \ldots\ldots\ldots\ldots \quad (13)$$

in which

$$T_r = \frac{c_r\, t}{a^2} \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

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[5] "The Conduction of Heat in Solids," by H. S. Carslaw and J. C. Jaeger, 2nd Ed., Oxford University Press, London, 1959, p. 199.

and $\pm \beta_n$ are the roots of the equation $J_o(\beta) = 0$, in which $J_o(\beta)$ is the Bessel function of the first kind. A. B. Newman has presented[6] numerical values for Eq. 13.

Otherwise,

$$U(T_r) = f_r(T_r) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

in which f_r indicates "a radial function of."

In the triaxial apparatus, an estimate of the degree of consolidation of the cylindrical sample is obtained by measuring the volume of water, v_t, forced out of the sample during compression to time t. From Eq. 8 it can be deduced that

$$U(T_r) = \frac{v_t}{v_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

for the radial case, in which v_p is the volume of water expelled from the sample at the conclusion of primary compression.

An equation similar to Eq. 9 can be written for the radial problem

$$\frac{U(T_r)}{U(N\,T_r)} = \frac{f_r(T_r)}{f_r(N\,T_r)} = \frac{v_t}{v_{N\,t}} \quad \ldots\ldots\ldots\ldots \quad (17)$$

and a plot similar to Fig. 1 can be prepared using the left-hand side of Eq. 17, values of N, and Eq. 13. Thus compression ratios can be obtained from the measurements of water volumes expelled at various times t and N t, and used with the radial plot similar to Fig. 1 to give the radial coefficient of consolidation.

The coefficients of consolidation for the case of both radial and vertical drainage can only be obtained when both are occurring simultaneously in a triaxial apparatus, if their ratio to one another is known.

The use of the method applied to the conventional consolidation test can best be demonstrated by an example.

EXAMPLE

The example used for purposes of demonstration is that given by Taylor.[7] From the information given therein, Table 2 can be prepared.

In Table 2, Cols. 1 and 2 show the data obtained from the diagrams in Taylor and Cols. 3 and 4 are arbitrarily chosen, convenient initial times and N-values from which the compression ratios shown in Col. 5 are obtained for an assumed corrected zero dial reading of 2097 as given by Taylor. If we choose the initial time of 4 min and N = 2.25 as an illustration, then the c_v computation is carried out as follows:

$$t = 4 \text{ min}; \; N\,t = 9 \text{ min}:$$

$$\text{Compression ratio} = \frac{2097 - 1815}{2097 - 1700} = 0.710.$$

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[6] "The Drying of Porous Solids: Diffusion Calculations," by A. B. Newman, Transactions, A. I. Chem. E., Vol. 27, 1931, p. 310.

[7] "Fundamentals of Soil Mechanics," by D. W. Taylor, John Wiley & Sons, Inc., New York, N. Y., 1948, pp. 239-242.

Entering Fig. 1 with this compression ratio and N = 2.25, we get T = 0.26. Taylor gives the average thickness of the sample as 1.51 cm so that we can use Eq. 6 to get

$$c_v = \frac{0.26 \times (1.51)^2}{4 \times 60} = 24.7 \times 10^{-4} \text{ sq cm per sec}$$

The other values are computed in the same way and are shown in Col. 6.

It will be seen that the values of the coefficient of consolidation are initially larger than they are at greater times, a fact which, according to the previous exposition, is due to the incorrect choice of initial dial reading. If we take a corrected zero dial reading of 2093 instead of the 2097 computed by Taylor, we find that the coefficients of consolidation computed on the basis of the new compression ratios, as shown in Cols. 7 and 8, are more consistent as the initial times increase. It would, therefore, appear that the soil on which the test was carried out had a coefficient of consolidation of about 23 x 10^{-4} sq cm per sec the corrected zero dial reading was about 2093.

TABLE 2.—EXAMPLE OF COMPUTATION OF COEFFICIENT OF CONSOLIDATION

Time, in minutes (1)	Compression dial reading (2)	Initial time (3)	N (4)	Corrected Zero Dial = 2097: Compression ratio (5)	Corrected Zero Dial = 2097: Coefficient of Consolidation, in x 10^{-4} (6)	Corrected Zero Dial = 2093: Compression ratio (7)	Corrected Zero Dial = 2093: Coefficient of Consolidation, in sq cm per sec x 10^{-4} (8)
0	2125						
0.25	2025						
1.0	1953	1.0	4	0.511	36.1	0.504	22.8
2.4	1882						
4.0	1815	4.0	2.25	0.710	24.7	0.708	23.8
6.0	1750						
9.0	1700	9.0	2.22	0.824	22.8	0.822	22.6
15	1638	15	2.00	0.912	21.5	0.910	21.3
20	1615						
30	1593						

The effect of secondary compression is to indicate greater compressions at the larger times than those which would occur were primary compression or consolidation the only phenomenon occurring. Consequently, the compression ratios at large times are smaller than the values which would be obtained on the basis of consolidation alone. This leads to smaller time factors from Fig. 1 and logically, to smaller c_v-values. This effect is evident in both Cols. 7 and 8 in Table 2. Table 2 also demonstrates, however, that the c_v-values obtained at medium to large times, being less sensitive to the corrected zero dial reading, are closer to the "real" value for the soil under the test conditions.

A good method of using Fig. 1 therefore is to use "medium" times (in the region of 2 min to 10 min for many soils under the test conditions described by Taylor) to compute a first coefficient of consolidation with an assumed zero dial reading which can then be applied to the dial readings at shorter times through Fig. 1 to recompute the zero dial reading. This can be repeated until consistent values of the coefficient of consolidation are obtained. The procedure is rapid in practice.

Finally, Fig. 1 can be used to obtain the amount of primary compression which has taken place at any time. For example, at initial times of 4 min and 9 min, from Table 2, we can see, on entering the U(T)-curve of Fig. 1 with the appropriate compression ratios and N values, that the sample is 0.56 and 0.78 consolidated, respectively.

CONCLUSIONS

A new method has been presented for the determination of coefficients of consolidation in soils from the volume changes which take place during the consolidation process. The actual process of consolidation from which the results are obtained is not necessarily the usual one-dimensional flow situation, but may involve radial flow or flow in several dimensions provided an analytical or numerical solution to the problem has been obtained.

The method is rapid and convenient in use, being adaptable to the determination of one or many values from one test, depending on the number of readings obtained or desired. The readings need not be taken at prescribed intervals. In use, the method is self-checking.

APPENDIX.—NOTATION

The following symbols have been adopted for use in the paper and are presented herewith for the guidance of discussers:

a = radius of cylindrical sample;

c_r = coefficient of consolidation for radial drainage;

c_v = coefficient of consolidation for drainage in the z-direction;

d_p = compression dial reading at end of primary compression;

d_s = corrected zero compression dial reading;

d_t = compression dial reading at time t;

$d_{N\,t}$ = compression dial reading at time N t;

e = base of natural system of logarithms;

$f(T)$ = function of T;

H = characteristic sample dimension;

$J_o(\beta)$ = Bessel function of of first kind and zero order;

M = summation term = $\frac{\pi}{2}(2\,m + 1)$;

m = integer of summation;

N = ratio of times or time factors, a dimensionless number;

n = integer of summation;

Δp = initial pressure increment;

r = radial distance;

T = time factor defined in Eq. 6;

T_r = radial time factor defined in Eq. 14;

t = time;

U = average degree of consolidation;

$U(T)$ = average degree of consolidation at time factor T;

$U(N\ T)$ = average degree of consolidation at time factor $N\ T$;

u = excess over hydrostatic pore water pressure:

$\bar{u}_t$ = space average value of u at time t;

$u_{z,t}$ = excess over hydrostatic pore water pressure at depth z, at time t;

v_p = volume of water expelled from triaxial sample up to end of primary compression;

v_t = volume of water expelled from triaxial sample up to time t;

$v_{N\ t}$ = volume of water expelled from triaxial sample up to time $N\ t$;

z = distance along z-axis; and

β_n = roots of equation $J_o(\beta) = 0$.

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DISCUSSION

Note.—This paper is a part of the copyrighted Journal of the Soil Mechanics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. SM 1, February, 1961.

TRACTIVE RESISTANCE OF COHESIVE SOILS[a]

Closure by Irving S. Dunn

IRVING S. DUNN,[1] M. ASCE.—The comments of Mr. Masch are very interesting and constitute a valuable addition to the original paper. Mr. Masch's description of the scour rate index and erosion characteristics of the soil, along with the recommended jet geometry, appears to be the result of extensive research, and a more complete publication is awaited with anticipation.

Mr. Sherard has also presented valuable new information in his discussion of the effect of soil properties on piping in earth dams. The following comments are in answer to his questions.

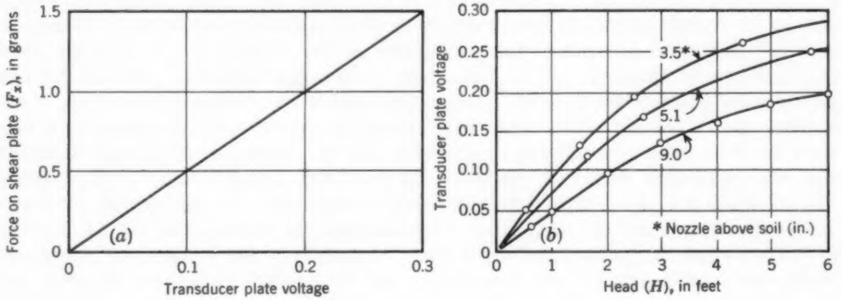

FIG. 1

1. The experimental approach to cohesive soil erosion using the submerged jet is the only approach that has been used by the writer. However, other researchers have used rectangular flumes and flumes with varying cross section designed to produce constant tractive forces. At the present time, a Hydraulics Division task force is being organized to study erosion of cohesive soils, and their report will undoubtedly summarize the different approaches that have been used.

2. Densities of the remolded soil samples were not measured. The resistance of the sample to vane shear was designed to be within the same range of values as that exhibited by the soil in place on the canal beds, and this resistance was produced using very low pressures. An investigation of the erosion

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[a] June 1959, by Irving S. Dunn (Proc. paper 2062).

[1] Prof., Civ. Engrg. Dept., Colo. State Univ., Fort Collins, Col.

characteristics of more compact soils, using the same experimental methods, would be in order to extend the data and make the results valid for soils in earth dams.

3. In the calibration of the jet test, the measurement of the small forces on the steel plate presented only one difficulty; the electrical measurements had a tendency to drift with time, and tests were repeated many times to eliminate the effect of the drift. Calibration curves are shown in Fig. 1. The shape of the plate influences the force, and the curves must be modified for any change in the geometry.

4. The opening around the shear plate in the jet calibration test was 0.03 in. and was filled with water. It is felt that the flow in the boundary layer above the plate was disturbed little by this discontinuity.

5. T_c is defined in this paper as the critical tractive stress in the laboratory experiments. As such, it includes a contribution to shear strength of the soil caused by a vertical change in momentum of the flowing water. This contribution is very small, and $T_c = S_h +$ (contribution). (S_h is tractive resistance).

RATE OF CONSTRUCTING EMBANKMENTS ON SOFT FOUNDATION SOILS[a]

Closure by Herbert L. Lobdell

HERBERT L. LOBDELL,[1] M. ASCE.—The writer wishes to extend his appreciation to Messrs. Gotolski, Cedergren, Silva, and Gilbert for their interest and comments on this paper.

Mr. Gotolski speaks of measuring pore water pressure during consolidation tests under various loads and degrees of consolidation and then performing shear tests at the same applied loads and degrees of consolidation. From this, he says, a relationship between pore water pressure and strength may be obtained. The pore water pressures have been measured at the base of consolidation test specimens in research studies[2,3] in which it was desired to check the consolidation theory, but the writer fails to see how such results can be applied to shear tests. As part of a research study,[4] the writer performed a series of triaxial compression tests in which the total lateral pressure σ_3 was kept constant for all tests, but in which the percentage of consolidation was varied, as Mr. Gotolski apparently suggests; pore pressure measurements were made in the middle of these specimens. From time-consolidation plots and pore pressure measurements made from a previously run fully consolidated triaxial specimen $(Q_c$ test$)$, it was possible to predict approximately at what point to stop consolidating the samples in order to reach a desired average over-all percentage of consolidation before loading to failure. As might be expected, as the percentage of consolidation increased, the strength increased. However, because of the variation of pore pressure throughout the samples at the time of loading to failure, it was concluded that no practical application could be made from, this particular series of tests. It is believed that all relationships between pore pressure and strength must be founded on the basic effective shear strength relationship $\tau = \bar{\sigma} \tan \phi_e = (\sigma - u) \tan \phi_e$, the terms of which are defined in the paper.

In response to Mr. Gotolski's statement that "the allowable pore pressure varies with the height of embankment; the higher the embankment the lower the allowable pore pressure," the writer wishes to point out the pore pressure divided by the weight of fill ratio beneath the full height of embankment that was worked out for the hypothetical problem shows a generally increasing trend as the height of fill is raised. Such ratios may be what Mr. Gotolski refers to when he speaks of maintaining the pore water pressure at a predetermined value during filling. However, from the range of these ratios shown, and the obvious fact that the pore water pressure is non-uniform both vertically and laterally beneath the slopes during filling, the writer is prompted to wonder which predetermined value of the numerous possible values of pore pressures Mr. Gotolski refers to.

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[a] October 1959, by Herbert L. Lobdell (Proc. paper 2214).

[1] Soils Engr., Woodward-Clyde-Sherard & Assoc., Greer Engrg. Assoc. Div., Montclair, N. J.

[2] "Research on Consolidation of Clays," by Donald W. Taylor, M. I. T., Publications from the Dept. of Civ. and Sanitary Engrg., August, 1942.

[3] "Consolidation of Clay, with Special Reference to Influence of Vertical Sand Drains," by Suen Hansbo, Proceedings, Swedish Geotechnical Inst., No. 18, Stockholm, 1960.

[4] "Shear Strength of a Clayey Silt," by Herbert L. Lobdell, thesis presented to Rutgers Univ., New Brunswick, New Jersey, in 1955 in partial fulfillments of the requirements for one degree of Master of Science.

The writer welcomes the remarks and analysis with its detailed step by step procedure contributed by Mr. Cedergren. In regard to the subject of shear strength and various of angles of friction that have been used in the past, the writer believes it is sufficient to say that effective stress concepts have gained widespread acceptance in recent years; that appears to be borne out by the numerous presentations and papers contributed at the recent ASCE Research Conference on the Shear Strength of Cohesive Soils held at Boulder, Colorado 1960.

Mr. Cedergren's method of establishing a safe rate of filling appears to be basically the same as that presented by the writer in that the fill is broken into increments, and that the effect of each lift on the over-all consolidation of the foundation soil is computed. It is gratifying to hear that one of long experience in such problems has judged the necessary and time-consuming analysis required to predict rate of filling to be warranted, and that he obviously considers such analyses to be of substantial value. The writer regrets, however, that Mr. Cedergren did not explain how he copes with the problem of variation of stress distribution and shear strength along potential failure planes.

The writer agrees with Mr. Silva that the analysis could be improved further by introducing a different value of coefficient of consolidation for each increment of load. However, this involves a separate time-consolidation curve for each step, and it is believed that from a practical standpoint there is a limit to the amount of time that can be spent on analyzing such a problem, particularly when so many variables and unknowns are involved. The writer has recently observed in published curves, that where the c_v plot does follow a definite trend similar to that shown in Mr. Silva's Fig. 1, instead of being scattered, the minimum c_v value is frequently in the general range of the pre-consolidation point. The reason for an increase in c_v in these curves beyond that point, whereas at the same time there is presumably a decrease in permeability, is not readily apparent to the writer.

Mr. Silva's method of determining radial permeability by means of a porous central drain in a standard consolidation test represents a novel and reasonable approach. Similar tests reported by Hansbo[3] indicate that results using such central drains agree fairly well with those obtained by consolidation of slices of clay cut out in a vertical direction so that the pore water escape is parallel to the clay strata.

When Mr. Silva asks for the writer's viewpoint regarding extending the method to the case shown in Fig. 3 of Mr. Silva's discussion, the writer believes that question is answered in the paper under the heading "Design Procedure." where the total strength is expressed as the sum of the initial or natural strength and the sum of strength increases due to embankment loading.

While the single tube type of piezometer does have the advantage of simplicity, it has the disadvantage of being a target for construction equipment and it must be periodically extended above the fill, frequently to great heights. A double-tube closed system can be extended outside the slope of an embankment where no extensions are necessary. It has been found that gas bubbles can be troublesome in inorganic soils, as well as in organic materials, even in single tube systems.

The writer disagrees with Mr. Gilbert's statement that the writer has made several oversimplifications that might lead to dangerous conclusions by the casual reader. In the first place, because of the complexity of the problem, some averaging and approximating must be done in an analysis of this type in order to be applied to practical problems; secondly, the writer believes that this is hardly a topic to be treated in a casual manner.

For various reasons it is difficult to answer some of Mr. Gilbert's statements. He states that the strength value obtained from the effective strength envelope represents the potential strength available after complete dissipation of pore pressure induced by increment of embankment loading. Examination of the basic shear strength equation shown previously (which the writer has employed to establish a relationship between effective overburden stress and shear strength) indicates that the shear strength can be essentially determined for any combination of total normal stress and pore pressure and that pore pressure does not have to be dissipated. Mr. Gilbert then adds that the strength of interest for stability, however, is the strength immediately available at the instant of incremental loading and that ignoring the increase in pore pressure will lead to a safety factor very much on the unsafe side. Once again, examination of the basic shear strength equation will include that increase in pore pressure, if applied properly, is not ignored.

Mr. Gilbert states that the one main advantage to the effective stress approach to embankment stability is direct correlation between available shear strength at any time and the observed pore pressures at that time. Later in the same paragraph he appears to be inconsistant when he states that the large number of piezometers needed at any cross-section to determine accurately the pore pressure variation along potential failure surfaces make the advantage of the effective stress concept more theoretical than practical. The writer believes that it is more important to install several piezometers beneath a typical slope or section, or beneath zones considered critical, than to install them at regular intervals beneath the center line and toe of slope of an embankment. For example, on a sand drain project that the writer was connected with, eight out of a total of 14 piezometers were concentrated at various points beneath one slope, which was considered the most critical because of the depth of soft foundation soils.

[illegible]

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[illegible]

DESIGN OF UNDERSEEPAGE CONTROL MEASURES FOR DAMS AND LEVEES[a]

Closure by W. J. Turnbull and C. I. Mansur

W. J. TURNBULL,[1] F. ASCE, and C. I. MANSUR,[2] F. ASCE.—The purpose of the paper was to demonstrate some of the general principles applicable for control of underseepage through specific experience and application. However, as Mr. Cedergren points out in his discussion, the information contained in the paper is of little value unless it is applied in designing new levees or in reviewing the safety of existing levees. The review of the safety of existing levees is important, because many of the levees have not been subjected to design flood stages nor has their safety regarding underseepage been adequately investigated.

Mr. Cedergren presents an interesting example of the design and use of a toe drain for controlling seepage through and beneath a levee. As he pointed out, it is most important that such a toe drain penetrate an adequate depth into the principal water-carrying stratum and that the filter or drain have adequate capacity for carrying the seepage flow.

It is believed that Mr. Suter has misinterpreted the writers' paper to the extent that it claims that the seepage problem necessarily comes from the riverside of the levee based on "known seepage laws." Instead of this, the treatment in the paper is for seepage coming from the riverside because this is known to be the case in the rather broad, flat valley areas of the Lower Mississippi Valley with which the writers have had considerable experience. The special situation in which seepage may be coming predominantly from the landside or both landside and riverside was noted in the writers' closure to the first paper of this series of three papers.[3] The writers cannot agree with Mr. Suter that riverside blankets are rather useless. The covering of exposed foundation sands between the levee and the river bank, in which the river bank is some distance from the levee, with an impervious blanket will materially reduce underseepage flow and pressure landward of the levee. In large and deep rivers, the use of an impervious blanket in the river itself is usually not practical. Neither can the writers agree that relief wells are of no benefit in controlling seepage beneath levees founded on pervious foundations or that they are only of negative benefit. No instance is known in which properly designed and installed pressure relief wells have damaged any structures. For the condition discussed by the writers, namely, with the basic source of seepage from the riverside, the relief measures, regardless of what type, should be located adjacent to the landside toe. The writers cannot agree with Mr. Suter that the fundamental principle of seepage control is to cause the formation of sand boils to be located as far away from the levee as possible rather than to avoid their formation.

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[a] October 1959, by W. J. Turnbull and C. I. Mansur (Proc. paper 2217).

[1] Chf., Soils Div., Corps of Engrs., Waterways Experiment Sta., Vicksburg, Mass.

[2] Engr., Hwy. Constr. Div., Fruin-Colnon Contracting Co., St. Louis 3, Mo.

[3] "Closure to Investigation of Underseepage—Mississippi River Levees," by W. J. Turnbull and C. I. Mansur, Proceedings, ASCE, October, 1960, p. 121.

The discussion by Messrs. Bitoun and Christiansen points out a very neat solution for a case of undesirable seepage caused by two opposing gradients, one the rather steep sloping groundwater table toward the river and the other the seepage flow from riverside of the levees to the landside produced by the flood waters. At this location, it appears that the ground surface beginning at the river bank rises at the rate of about 1 ft in 100 ft, which is also about the slope of the groundwater gradient toward the river. The valuable lesson to be learned from the material presented by the discussers is that the excellent solution arrived at would not have been possible without the complete knowledge of the physical conditions existing below the ground surface at the site.

Mr. Evans has presented an interesting discussion on the relationship between the safety factor computed on the basis of critical gradient and the safety factor computed on the basis of uplift. Mr. Evans mentions the formula for critical gradient, but it is believed that the formula is misstated, probably as a result of typographical error. The denominator of the right-hand portion of the equation should be $1 + e$. Mr. Evans points out that the formula is found in numerous textbooks, that is correct. Other textbooks refer to the critical gradient as being equal to the ratio of the submerged unit weight of the soil and weight per cubic foot of water. Either equation gives the same numerical results.

Mr. Evans points out that for a gradient safety factor greater than 1, the uplift safety factor is also greater than 1, and that the uplift safety factor is less than the gradient safety factor. This is true. Mr. Evans might also have pointed out that for a gradient safety factor less than 1, the uplift safety factor is also less than 1, and the uplift safety factor is greater than the gradient safety factor, that is just the reverse of the previously stated relationship. These relationships are true for the condition of 100% saturation. For conditions less than 100% saturation, the relationship changes as mentioned by Mr. Evans.

If a vertical line is drawn (Mr. Evans' Fig. 1D) at the ratio of the gradient safety factor to the uplift factor equal to 1, it intersects the critical gradient curves at different ratios of blanket gradient. The significance of the interpretation of this line is that when the blanket gradient is equal to the critical gradient, then the ratio of the gradient safety factor to the uplift safety factor is equal to 1 and each safety factor is equal to 1. On the right side of this line the ratio of safety factors is greater than 1 and the blanket gradient is less than the critical gradient. To the left of this line the ratio of the safety factors is less than 1 and the blanket gradient is greater than the critical gradient.

The preceding discussion is somewhat academic without considering the actual conditions in the field. Critical conditions and failure brought about by underseepage are usually one of the following: (a) piping as a result of seepage through relatively low cohesive to cohesionless materials such as silts and sands, (b) piping of material through holes in the blanket which are produced probably by burrowing animals or decaying roots, etc., and (c) abrupt piping as a result of a blowout in a relatively tight clay blanket. Of the preceding three, the most common in the writers' experience have been the first two listed.

Mr. Evans rightfully points out that the critical gradient is probably more applicable to cohesionless or relatively cohesionless materials because in these materials at least some measurable seepage passes through the blanket and piping failure is the most likely type of failure to expect. He points out that for relatively tight clay blankets, the factor of safety computed by the uplift procedure is more applicable. The writers certainly do not disagree with this concept where the ratio of the gradient safety factor to the uplift safety factor is greater than 1, particularly when the percentage saturation is less than 100. However, where this ratio is less than 1, the writers believe it would be more conservative to use the critical gradient. Mr. Evans might argue against this and say at this point the blanket has already failed. The writers, however, feel that in the case of cohesive clay blankets, particularly in ditch bottoms in which the span is relatively short, the blanket might be sufficiently tough and cohesive to hold a pressure somewhat greater than the critical. If this is true, the critical gradient safety factor would certainly be more conservative.

FOUNDATION VIBRATIONS[a]

Discussion by I. Alpan, Joseph G. Perri, A. A. Eremin

I. ALPAN.[4]—The author has contributed a comprehensive and significant paper in a field that has hitherto received comparatively little attention from soil engineers, namely that of soil dynamics as applied to machine foundations.

The prediction of the dynamic response of machine foundations at resonant and operating frequencies is presented by a method derived from previous theoretical work.

Some of the points raised seem, however, to require further clarification.

The soil underlying the machine foundation is considered as a semi-infinite elastic medium in which waves, emanating from a point source, are propagated (the dynamic equivalent of the Boussinesq problem).

However, serious theoretical objections have been raised to the possibility of resonance in such a medium. For example, it was shown by K. Polz (39)[2] that the occurrence of natural frequencies is not possible in a semi-infinite medium unless it contains rigid surfaces from which the propagated waves are reflected back to the exciting source. Similarly, D. G. Christopherson, in the discussion on a paper on machine foundations (40), makes the point that in order to establish natural frequencies a characteristic distance of the medium is required. Indeed, the natural frequency of an elastic body is given by the relation (41):

$$f_n = \frac{c}{L}\sqrt{\frac{E}{\rho}} \qquad \text{.....................(21)}$$

in which c is a constant, L a length, E an elastic modulus and ρ the mass density of the medium.

The case of an elastic layer resting on a rigid medium is briefly considered. The author quotes results of presumably analytical investigations that show that if an oscillator acts on such a layer the ratio of layer thickness to the radius of the oscillator base greatly influences the amplitudes. For a ratio larger than 6, however, the response is that of a semi-infinite medium.

It would be of interest to know how this ratio affects the natural frequencies. Model experiments have been carried out by W. Eastwood (42) in which the thickness of a sand layer over a rigid steel plate varied between 0.67 and 3.33 of the base radius and no change in the natural frequency was measured. It should be mentioned that in the tests in question impact was used.

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[a] August 1960, by F. E. Richart, Jr. (Proc. paper 2564).

[4] Sr. Lecturer, Israel Inst. of Tech., Haifa, Israel.

[2] Numerals in parentheses, thus (39), refer to corresponding items in the Supplementary Bibiliography.

The paper does not enter to any extent into the problem of damping. There seems to be some connection between the dimensionless quantity b as given by Eq. 10 and the concept of "system-damping" as developed by G. Ehlers (43) who considers damping as being due to "energy radiation" into the foundation soil. Polz also uses this concept in his work (39): it should, therefore, prove of considerable interest to examine this point further.

It should be possible to determine the damping factor of a soil from amplitude versus frequency curves such as those presented in numerous publications by Lorenz (44), (45), (46). Considering conditions at resonance, the phase diagram of the forces involved permits the evaluation of the damping constant. This analysis applies, of course, only to a system such as shown in Figs. 3(a) or (b). For a system with a non-linear spring, this approach would be only approximately valid due to the pseudo-harmonic nature of the vibrations.

Experimental evidence shows that damping increases with the exciting forces (45) and the contact area of the foundation (47)(48). It seems, therefore, that the evaluation of amplitude versus frequency curves obtained with experimental oscillators would offer the most reliable basis for predicting the dynamic soil response.

The non-linear behavior of soils, as determined by oscillator tests, is questioned by the author, his argument being that the soil may have been compacted by the vibratory action and possible "jumping." However, the curves presented by Lorenz (44)(45) show the soil to have a sub-linear, "soft spring," characteristic. Surely the effect of compaction would have been to "harden" the spring, that is, to influence the soil characteristic in the direction of linearity.

The case most often encountered in practice is that in which the magnitude of the periodic forces varies with the square of the frequency. For this case, the "weightless-subgrade procedure" is shown to give results that agree closely with those obtained by the method presented in the paper, as concerns the resonant frequency. Damping is of little influence as shown by the following easily derived expression:

$$f_{nd}^2 = \left(1 - D^2\right) f_n^2 \qquad (22)$$

in which f_{nd} is the damped natural frequency, D the damping ratio, and f_n the undamped natural frequency. A typical value would be D = 0.15 to 0.2 (46). As damping is neglected in the "weightless-subgrade procedure" it is, of course, not possible to obtain the value of the resonant amplitude by this method.

The writer's comments on this point may, perhaps, best be illustrated by a few computations:

If a value of D = 0.2 of the damping ratio were used with Fig. 3(b) and the data of Example A, the resonant amplitude (at 760 rpm) is found to equal 0.0031 in.

Now Fig. 3(b) shows that for frequency ratios of two or more the amplitude approaches the value of $W_1 \left(1/W_o\right)$, the "eccentricity factor" of dynamic theory, that equals 0.000625 in. for Example A. This value compares quite well with 0.00074 in. found by the author's method for 1800 rpm, that is, a frequency ratio of 2.37. The value found by this method for the resonant amplitude was 0.0007 in., that is 4.4 times smaller than the value of 0.0031 in. computed previously by the weightless-subgrade procedure using a fairly high value of damping.

The fact that this method furnishes a slightly lower amplitude at resonance than at 1800 rpm, that is, 0.0007 as compared with 0.00074 in., is attributed by the author to the approximation involved. He does not, however, take exception to the fact that the amplitude at frequency ratios of 1.0 and 2.37 are of the same order of magnitude.

Measurements on actual foundations (49) show the response curves to be fairly similar to those corresponding to the weightless subgrade hypothesis, such as shown in Fig. 3(b), that is, at resonance the amplitudes are considerably larger than at frequency ratios of, for example, two and above. Therefore, in the example, a resonant amplitude of 0.0031 in. appears more likely than that found by the author. It would seem that predictions of resonant amplitude, based on the method under consideration, are on the unsafe side.

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JOSEPH G. PERRI,[5] F. ASCE.—Many engineers eminently qualified by knowledges and experience for the design of foundations will always welcome new theoretical and experimental information on the subject because of their ability to select and screen the most useful part of it. The same engineers however, will naturally object to the idea that the same information can be used by engineers who have only an elementary basis of the theory of vibrations and who believe that is is sufficient to be acquainted with the short frequency formula, $f_n = \frac{1}{2\pi}\left(\frac{k\,g}{W}\right)^{1/2}$.

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[5] Staff Consultant, Burns and Roe, Inc., Engrs. and Contractors, N. Y.

The writer's objections to the indiscriminate use of some of the information given in the paper springs from the fact that extensive structural damage and production losses which have been caused in observed cases of poorly designed compressor foundations necessitate the need of pointing out the limits of applications of both theoretical and experimental formula for the design of machinery foundations.

Only very important reasons have made the writer deviate from his policy of shunning controversial discussions. About twenty years ago the writer was confronted with the task of supervising the design of many industrial plants. On interviewing the engineers who were available for the design of machinery foundation he was amazed to discover that the only thing they knew about it was that "by providing a concrete block sufficiently heavy under a machine there was no need of worrying about anything else." A few years later the writer was often asked to investigate the design of foundations under heavy compressors which were kept inoperative because of vibration troubles. The findings showed that massive concrete foundation blocks with mass ratios of even 1 to 9 to those of the machine supported by them were not sufficient to absorb the transmitted vibrations because of their high center of gravity above their bases and the poor condition of the subsoil under them; which in some cases had been worsened by some machine oil seeping through it.

Between 1944 and 1945 the writer, while at the employ of an industrial engineering firm, conducted some research work on the subject of designing machinery supports and reciprocating engine foundations and, after having examined the literature available to him at that time, such as (50)(51)(52)(53) and many other references, some of which have been mentioned by the author, he developed methods and formulae for a more rationalized procedure. Restrictions and regulations imposed upon him by his employer's policy of keeping useful information within the limits of the office personnel prevented the writer from publishing the paper. Perhaps it was better that way because a literary use of the paper results by engineers unskilled on the subject might have led them to the inproper handling of the problem to be analyzed.

Before discussing to what extent one can trust the reliabilty of the soil constants required in the design of foundation blocks and what percentage of error in the final results with respect to vibration transmissibility one can allow, let us examine the basic mathematical criteria associated with the determination of the frequencies of the system. For any type of foundation similar to the one shown in Fig. 14 two simultaneous motions take place; translatory and rotatory motions. The horizontal and rotational modes are thus coupled and vibration in one of these modes cannot exist independently of vibration in the other mode. The coupled rotational and horizontal translatory motion of the foundation block are usually expressed as horizontal motion of its center of gravity and rotation of its mass with respect to the axis through its center of gravity. This complex type of motion may be envisaged physically as a simple rotation of the body about an imaginary axis remote from but parallel to the axis passing through the center of gravity of the body. In other words, no horizontal translation would be experienced by the gravity axis.

Consider the typical case of a foundation block subject to periodic horizontal force, F cos ωt, acting e-ft above the center of gravity of the block and laying in a vertical plane passing through the same center. The system (see Fig. 15), is symmetrical with respect to the two normal vertical planes passing through the mass center. It may be thought of rotating about an imaginary axis passing

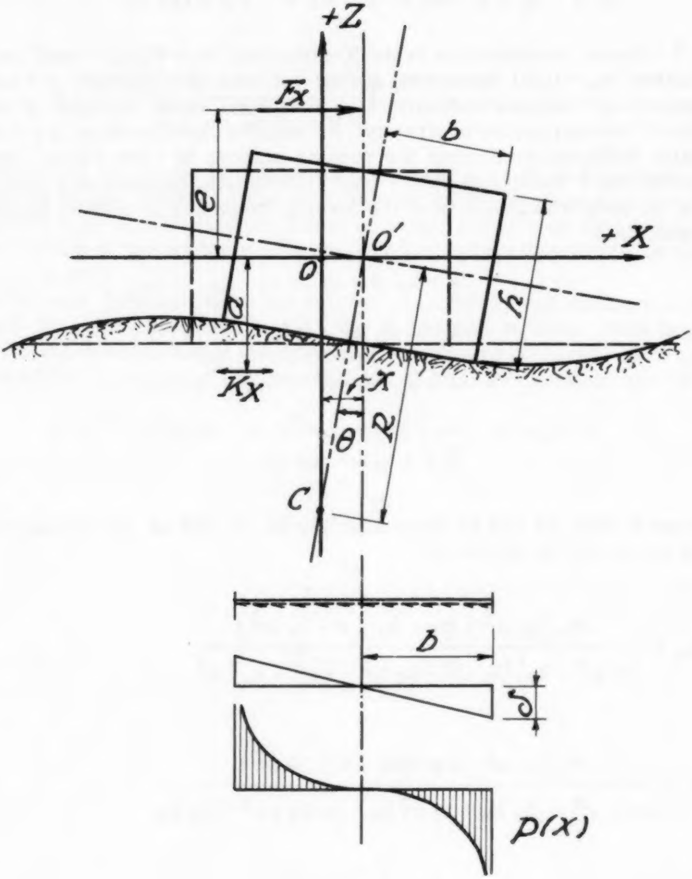

FIG. 15.—TRANLATION AND ROTATION

through c at a distance R below its center of gravity. If the origin of the coordinate is taken at the center of mass (X and Z being the axes laying in a plane parallel to the plane of the paper and Y the axis normal to it) and if x represents the total horizontal displacement of the mass center from the Z-axis due to translation and rotation, the equations of motion can be written at once as follows: Motion along the X-axis

$$m \ddot{x} = -k_x x + k_x a \theta + F_x \cos \omega t \dots \qquad (23)$$

Rotation about the Y-axis

$$I_y \ddot{\theta} = k_x a x - k_x a^2 \theta - k_T \theta - F_T e \cos \omega t \dots \qquad (24)$$

in which $\ddot{x}$ = linear acceleration in the X-direction; $m = W_o/g$ = total mass of the oscillator; k_x = total horizontal spring constant of subgrade; θ = angular displacement; ω = angular velocity; I_y = rotational mass moment of inertia with respect through center of gravity; $\ddot{\theta}$ = angular acceleration; $k_T = M/\theta$ = total elastic stiffness coefficient for rocking motion; M = the elastic moment of p(x) about the Y-axis; and p(x) = load distribution, function of x only. The motion of the body which occurs at the forcing frequency is defined by the following equations;

$$x = x_o \cos \omega t \dots \qquad (25)$$

$$\ddot{x} = - x_o \omega^2 \cos \omega t \dots \qquad (26)$$

$$\theta = \theta_o \cos \omega t \dots \qquad (27)$$

and

$$\ddot{\theta} = - \theta_o \omega^2 \cos \omega t \dots \qquad (28)$$

Substituting in Eqs. 23 and 24 from Eqs. 25, 26, 27 and 28 and solving simultaneously for x_o and θ_o we have:

$$x_o = \frac{F_x \left(k_x a^2 - k_T - k_x a e - I_y \omega^2\right)}{\left(m \omega^2 - k_x\right)\left(I_y \omega^2 - k_x a^2 - k_T\right) - {k_x}^2 a^2} \dots \qquad (29a)$$

$$x_o = \frac{F_x\left(k_x a^2 - k_T - k_x a e - I_y \omega^2\right)}{m I_y \omega^4 - \left(I_y k_x + m a^2 k_x - m k_T\right) \omega^2 - k_x k_T} \dots \qquad (29b)$$

$$\theta_o = \frac{F_x\left(m e \omega^2 - e k_x + a k_x\right)}{\left(m \omega^2 - k_x\right)\left(I_y \omega^2 - k_x a^2 - k_T\right) - k_x a^2} \dots \qquad (30a)$$

and

$$\theta_o = \frac{F_x\left(m e^2 - e k_x + a k_x\right)}{m I_y \omega^4 - \left(I_y k_x + m a^2 k_x - m k_T\right) \omega^2 - k_x k_T} \dots \qquad (30b)$$

The location of the remote axis through c is given by:

$$R\,\theta_o + x_o = 0 \tag{31}$$

$$R = -\frac{x_o}{\theta_o} \tag{32a}$$

and

$$R = \frac{a^2\,k_x - k_T - a\,e\,k_x - \omega^2\,I_y}{m\,e\,\omega^2 - e\,k_x + a\,k_x} \tag{32b}$$

The location of the center of rotation thus depends not only on the physical characteristics of the block and the soil but also on the position and frequency of the applied forces. The distance R may be either positive or negative depending on the position of the center of rotation which may be either below or above the center of gravity.

The natural frequencies in the coupled translatory and rotatory modes are obtained from the conditions that x_o and θ_o become infinite. This is possible only when the determinant of the system of equations approaches zero as a limit. Therefore by equating the denominator of Eqs. 29 and 30 to zero we have;

$$m\,I_y\,\omega^4 - \left(I_y\,k_x + m\,a^2\,k_x - m\,k_T\right)\omega^2 - k_x\,k_T = 0 \tag{33}$$

Let

$$I_y = m\,\rho_y^2 \tag{34}$$

$$\Omega_x^2 = \frac{k_x}{m} \tag{35}$$

$$\Omega_\theta^2 = \frac{k_T}{m\,\rho_y^2} \tag{36}$$

$$\Omega_x^2 = \frac{k_x}{k_T}\,\rho_y^2\,\Omega_\theta^2 \tag{37}$$

and

$$\Omega_x^2 = \eta\,\Omega_\theta^2 \tag{38}$$

By substituting Eqs. 34 to 38 into Eq. 33 in and rearranging terms

$$\left(\frac{\omega}{\Omega_\theta}\right)^4 - \left(\eta + \eta\,\frac{a^2}{\rho_y^2} - 1\right)\left(\frac{\omega}{\Omega_\theta}\right)^2 - \eta = 0 \tag{39}$$

This is a quadratic equation in $\left(\frac{\omega}{\Omega_y}\right)^2$. The roots of this equation are:

$$\left(\frac{\omega}{\Omega_\theta}\right)^2 = \frac{1}{2}\sqrt{\left[\eta\left(1+\frac{a^2}{\rho_y^2}\right)-1\right] \pm \sqrt{\left[\eta\left(1+\frac{a^2}{\rho_y^2}\right)-1\right]^2 + 4\,\eta}} \quad \ldots\ldots\ldots\ldots (40a)$$

or

$$\frac{\omega}{\Omega_\theta} = \frac{1}{\sqrt{2}}\sqrt{\left[\eta\left(1+\frac{a^2}{\rho_y^2}\right)-1\right] \pm \sqrt{\left[\eta\left(1+\frac{a^2}{\rho_y^2}\right)-1\right]^2 + 4\,\eta}} \quad \ldots\ldots\ldots (40b)$$

If Ω_c is substituted for ω, the two coupled natural frequencies, Ω_c, may now be determined in terms of the single natural frequencies, the dimensionless stiffness ratios, and the dimensions. Thus:

$$\Omega_c = \Omega_\theta \frac{1}{\sqrt{2}}\sqrt{\left[\eta\left(1+\frac{a^2}{\rho_y^2}\right)-1\right] \pm \sqrt{\left[\eta\left(1+\frac{a^2}{\rho_y^2}\right)-1\right]^2 + 4\,\eta}} \quad \ldots\ldots (40c)$$

This indicates that the Author's statement "The possibilities of resonance can be estimated for each conditions independently, and then the resulting motion determined by superposition" is apparently incorrect and therefore unacceptable. For the sake of correctess the writer points out that in the expression for I_o under the heading "Example B" there is an error. Thus $I_o = \frac{W_o}{3\,g}\left(h^2 + \frac{a^2}{2}\right)$ + etc. Should read $I_o = \frac{W_o}{3\,g}\left(h^2 + a^2\right)$ + etc.

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52. "Analytical and Experimental Methods in Engineering Seismology," by M. Biot, Proceedings, ASCE, January, 1942.

53. "Mathematical Methods in Engineering," by Th. Von Kármán, and M. A. Biot, McGraw Hill Book Co., Inc., New York, 1940, p. 404.

A. A. EREMIN,[6] M. ASCE.—In computing the pile foundation forces the author considered the stiffening effect produced by the pile tip bearing stresses. In the group pile foundation, the stiffening effect on foundation block will be increased by adding the effect of the neighboring piles.

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[6] Assoc. Bridge Engr., Bridge Dept., Calif. State Highways, Sacramento, Calif.

In analysis of stresses in the spread footing, Mr. Richart expressed the elastic properties of soil by the modulus of elasticity for shear. In the confined stresses, Mr. Richart recommended that the modulus of elasticity for shear shall be increased by 1/3. Computation of foundation stresses could be simplified by expressing the elastic properties of soil by the modulus of elasticity for direct stresses. In the case for the confined stresses, the modulus of elasticity for direct stresses may be obtained either from triaxial soil test or by oscillator test.

It would be interesting if Mr. Richart would comment on the effect of velocity of shifting of frequency of pulsating over the range of critical period of vibration in his closing discussion on dynamic forces in foundation block.

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PILE DRIVING ANALYSIS BY THE WAVE EQUATION[a]

Discussion by E. Jonas, A. A. Eremin, G. M. Cornfield, Robert D. Chellis
and Arthur F. Zasky

E. JONAS,[31] **M. ASCE.**—The author has rendered an important service to the civil engineering profession by providing a solution to the wave equation applicable to pile driving operations. His earlier paper[6] reached only a limited number of civil engineers and, therefore, did not rouse the interest and discussion that it deserved. By applying the theory of longitudinal stress transmission to pile driving problems, the author has introduced a rational approach to the analysis of pile driving action under a specified set of conditions.

For determining the stresses in the pile at any instant during impact, the author uses a series of simultaneous equations, that can be solved by an electronic computer. This method has certain advantages inasmuch as it can be adopted to any shaped pile, and the frictional and viscous resistance of the ground as well as the point resistance of the pile and the effect of hammer and cushion block can be included in the computations. The method has been used by the author mainly to determine the driving stresses in piles, although correlation of driving resistance to ultimate pile capacity has also been done.

In applying the theory of longitudinal stress propagation to the determination of pile capacity, it is implied that a compression stress wave is set up as a result of the impact between the ram and pile. This stress wave travels at a known speed toward the tip of the pile and is then reflected from the tip as a compression or tension wave depending on the ultimate ground resistance available immediately below the tip of the pile. According to the theory, if the available resisting force that the ground can develop is at least twice as large as the maximum force resulting from the compression stress wave, the entire stress wave is reflected and no permanent set occurs. In the case when the ultimate ground resistance is smaller than twice the peak stress at the front of the stress wave, part of the stress wave is reflected and the other part is absorbed by the ground. In no case can the reflected stress exceed one-half of the available ultimate ground resistance. On the other hand, the work performed during pile penetration is equal to the ultimate ground resistance times tip area of pile times permanent set. In accordance with these principles, it is believed that the quantities represented by $R\,u_m$ and R_p in Eqs. 13 and 16 should be used in the sense of "one half off the ultimate ground resistance" and not as the "ultimate ground resistance" as defined by the author.

It can be shown that in the "Illustrative Example" given by the author, equilibrium does not obtain at the end of the permanent set unless Eqs. 13 and 16 are modified as previously suggested. In this example, a 100 ft long 12 WF 53 pile was driven with a Vulcan No. 1 hammer against an assumed ultimate ground resistance of 200,000 lb. The permanent set under these conditions was computed as 0.20311 in. per blow. In other words, the pile if driven to a resistance of approximately 5 blows per in. would have developed the ultimate ground resistance of 200,000 lb. The maximum compressive force, F_{11}, occurring at the tip of the pile investigated was found by the author to be 405,000 lb. In the writer's opinion, in order to satisfy the equilibrium conditions at the tip of the pile when F_{11} becomes equal to 405,000 lb, the resistance of the ground must closely equal 405,000 lb and not 200,000 lb as stated by the author. The ultimate ground resistance could be slightly less than the value of F_{11} on account of the "ground quake" but a difference of 50% is difficult to accept.

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[a] August 1960, by E. A. L. Smith (Proc. paper 2574).

[31] Chf., Soil Mechanics and Foundations Engrg. Dept., Tippetts - Abbett - McCarthy - Stratton, New York, N. Y.

If R_u = 200,000 lb is considered as one-half of the ultimate ground resistance, equilibrium conditions would be satisfied at the tip of the pile. Accordingly, the ultimate pile capacity would be equal to 400,000 lb. As recommended by the author, a suitable factor of safety has to be applied to the ultimate pile capacity in order to arrive at the design pile capacity.

A method of applying the theory of longitudinal stress propagation for determining the ultimate pile capacity using energy concepts has been developed by John Lowe, III, and the writer.[32] This method, when applied to the pile and pile driving hammer given in the "Illustrative Example," of the paper, indicates that the ultimate pile capacity corresponding to a permanent set of 0.2 in. per blow is approximately 400,000 lb.

A. A. EREMIN,[33] M. ASCE.—The author stated that by using the conception of the wave equation and resorting to numerical integration on electronic computer, a solution of the pile driving problem can be obtained that produces accuracy with about 5%. Further, in his conclusion Mr. Smith stated that on account of various limitations in the field of soil mechanics and construction materials, an accuracy of foundation design within 5% is prevented.

In design of foundation precision of 5% is seldom considered as criterion. Considering of vibrating forces in pile driving, nevertheless, has important value even at the smaller precision.

In the numerical illustrative example, Mr. Smith considered velocity of stress distribution in the pile material same as velocity of sound distribution in similar solid material. Pile is, generally, driven in the conglomerated structure of soil material. Therefore, it should be expected that velocity of stress distribution will be retarded by various disturbances due to changes in physical properties of soil and pile.

The author considered that the constants of damping forces varied from 0.05 to 0.20. The similar damping constants are, generally, assumed in the vibrated single units. In pile driving, however, the damping forces may be increased by considering combined effect of tip bearing stresses, buckling stresses in pile, and effect of elastic properties of soil.

Mr. Smith stated that numerous numerical data was obtained from driving of piles by vibrating. Numerical information on driving long piles by vibrating would be highly appreciated.

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[32] "Capacity of Driven Piles Computed by Stress Waves," by J. Lowe, III, and E. Jones, (presented at the October, 1960 ASCE Annual Convention, Boston, Mass.

[33] Assoc. Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Pub. Wks. Bldg., Sacramento, Calif.

G. M. CORNFIELD.[34]—The many existing dynamic pile driving formulas are admittedly not very satisfactory as has been shown by several investigators. It is, however, suspected that the correlation between the formulas and test load data would have been improved if the investigators had eliminated certain cases to which it is known that dynamic formulas are not applicable, for example, pure friction piles in soft clays, and cases in which a re-drive test shows a larger set on redriving. Test loading is undoubtedly the best method of determining the ultimate resistance of a pile, but normally only a small percentage of piles on a site can be tested and there remains the problem of dealing with the remaining untested piles. Are these other piles to be driven to the same level as the tested pile, or to the same set, or to the same penetration into the main bearing stratum ? If the problem is to be solved using only borehole and soil test data as a basis, it has to be remembered that soils can be very variable in their characteristics even over short distances and, to make matters worse, cohesionless soils such as sands and gravels will have their properties altered by the driving of every pile on the site. Thus, the test loading of a few piles is but a partial solution and there is, therefore, a real additional need for some reasonably accurate method of making use of the driving data, that is the hammer energy, set, and so forth, to determine the ultimate resistance of a pile in appropriate cases. It is to be hoped that the application of computers to the solution of the wave equation will fulfull this requirement as well as the case of small contracts for which testing loading may be an uneconomical proposition.

It would appear that there are two possible ways of applying the results of computer analyses of the wave equation. The first and obvious one is for the determination of the ultimate driving resistance of piles. However, it may take a relatively long time before sufficient comparisons of wave equation computations, test load data, and soil information will have been made to confirm that the method gives correct results, that the right constants are being used, and so on. A second type of application of the wave equation would be to use it to study possible trends in the pile driving process, for example, the significance of each of the parameters and whether the value of the ultimate resistance is sensitive or not to particular ones. This can possibly be studied without much further delay on the basis of such wave equation computations that have already been carried out. Mr. Smith, has shown[35,36] examples of possible trends of the nature just mentioned.

Wave equation computation suggest[35] that for steel bearing piles of normal weights the effect of variation in pile length may be insignificant in which the lengths are over about 50 ft. Thus, two steel piles of the same weight per foot, driven with the same hammer, and to the same set per blow may have approximately the same ultimate resistance, even though the driven lengths of the piles are 50 ft and 100 ft respectively. This has led the writer to compare the failure test loads of a number of long piles with the computed driving resistances obtained by the Hiley pile driving formula[2] as he has not had the opportunity to use a computer. Eleven concrete piles (lengths 65 ft to 100 ft) and twenty-six steel piles (lengths 80 ft to 190 ft) were examined, and it was found that improved correlation as between the Hiley formula and the failure test loads resulted if, instead of using the actual pile length, an assumed artificial length of 50 ft was used in applying the formula for each case.

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[34] M. Sc., M. I. C. E., British Steel Piling Co., Ltd., London, England.

[35] "What Happens When Hammer Hits Pile," by E. A. Smith, Engineering News Record, September 5, 1957.

[36] "Pile Calculations by the Wave Equation," by E. A. Smith, Concrete and Constructional Engineering, London, June, 1958.

A diagram illustrating the application of the wave equation to steel bearing piles of various weights per foot, but of the same length and driven with the same hammer, to the same ultimate resistance is presented elsewhere.[36] A significant trend that is apparent in this case is that over a wide range of pile weights, that is from about 60 lb to 300 lb per ft of length, the set required lies within the narrow range of 5 ot 7 blows per in. This indicates that the pile weight is a relatively unimportant factor, for this particular example.

It is interesting to note that the preceding examples of trends arising from the application of the wave equation suggest the possibility that the ultimate driving resistance of piles may not be very sensitive to variation of the weight per foot run of the piles and of the pile lengths for a given hammer drop (fall) and set. The most significant factors apparently influencing the ultimate resistance would then be the hammer ram weight and equivalent drop (fall) as well as the set. It is the writers' hope to examine this in more detail by making use of the results of a large number of failure test loadings of concrete and steel piles.

The writer looks forward to hearing more about the wave equation and in particular to seeing published data, possibly in the form of tables as mentioned by Mr. Smith in another publication.[35]

ROBERT D. CHELLIS,[37] F. ASCE and ARTHUR F. ZASKEY.[38]—The valuable paper by Mr. Smith on a mathematical solution of the longitudinal wave equation for determination of pile driving resistances represents a great deal of painstaking study with consideration of the many variables, their effects, and methods of handling them. Availability of such a method of determining driving resistances may enable use of its results as a yardstick against which results by presently used formulas may be compared, as the author suggests. Load test results have been used as yardsticks, but they often are not available until piles or driving rigs are on the site for the start of driving, particularly for medium and small sized projects. Moreover, a load test result applies only to a particular set of conditions. Soil mechanics has provided methods for determination of soil capacities to carry loads applied by piles, such as (a) the sum of bearing and friction values determined by judgment, soils laboratory tests, or filed tests of soils in place, and (b) total static soil load carrying capacities by the cylindrical pier method. Use of the wave equation may make possible a more accurate determination of probable driving resistance at an early date.

The author wisely limited the scope of his paper at this time to presentation of the wave equation solution. This discussion comments on this method and also considers its possible use in practice. Relationships between the proposed method and present dynamic formulas and load tests are considered to aid in determining degrees of accuracy obtainable and expectable.

Computer programs such as the one described by Mr. Smith produce results that can not, as a practical matter, be attempted by hand. Therefore, disclosure of the detailed meaning of such procedures in a field of engineering must be relegated to those who possess the program. Development of such a program will cost several thousand dollars. Before making this investment, an engineer will want to compare results of finite difference computations to those of conventional methods, such as the Engineering News and Hiley formulas.

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[37] Struct. Engr., Stone & Webster Engrg. Corp., Boston, Mass.

[38] Systems Programmer, Nuclear Projects, Stone & Webster Engrg. Corp., Boston, Mass.

The familiar "engineering formula" must, for brevity, be summary or synoptic. It is contrived by artful omission. (Sequence of more and more omissions and the meanings and effects appear elsewhere.[39]) The Hiley formula is numerically frugal, refers where possible to observables, such as permanent set, and rebound, and accepts the discipline of budgeting the energy of hammer. The Engineering News formula is even more frugal of terms and is too simplified for the broad ranges of modern hammers and piles; for instance, it has no terms to express such wide variables as pile length or weight.

By contrast, the finite difference procedure develops a detailed impluse-momentum model. It is not summary or synoptic; it mimics the motion and stress of the pile.

For one process of computation to produce better results than another, it must utilize more information, or better information, or else make better use of the information that it has. A synoptic formula may merely omit from consideration information that is not available anyway. Opposing this is the possibility that engineers have failed to develop information for the reason that common methods are not set up to make use of it.

Comprehension of the principles of soil mechanics is needed when considering applicability of the wave equation, as well as of any pile-driving resistance formula or other method for obtaining load carrying capacity. Use and interpretation of such methods still remain partly science and partly an art. This is one of the principal reasons why the writers have advocated the use of the Hiley formula, backed by static investigations of end bearing and side friction. These methods require thought and consideration of the variables affecting driving and static bearing capacity and develop judgment or a "feel" for the relative effects.

The author mentions that, after a pile has been driven, the soil may largely retain its original supporting value of it may "set up" or "relax" around a pile. Also, effects of negative friction from gradual pickup of additional load on a pile from firm upper strata overlying incompletely consolidated soft strata may occur and continue with time. This action is not evaluated by dynamic or wave equations, but must be considered from the standpoint of soil mechanics, because lone periods of time are involved.

The author defines over thirty terms, many of which are variables, whereas others vary only from case to case. Such a term as "J," the damping coefficient, can also vary with soil stratification and effects of pile taper. The damping resistances are only in action during driving and do not carry permanent load. Some uncertainty surrounds the value of "E" in reinforced concrete piles, as it varies with mix, degree of curing, and age. Electronic computers have the ability to handle the mass of cases resulting from so many terms. Determination of a method of presenting this information requires study.

Comments on Author's Conclusions.—Conclusion 1 - The author states that the proposed method gives permanent set per blow, as well as instantaneous stresses for any specified conditions. The terminology of "permanent set" may be ambiguous and might mislead if the use of the work "permanent" is referred to mean that the method gives the long-term static load that can be carried without further settlement. The intent of the conclusion is assumed to mean that "permanent set" is the net penetration per blow (plastic displacement, that is the irreversible portion) remaining after elastic rebound from that blow has occurred.

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[39] "Pile Foundations," by Robert D. Chellis, McGraw-Hill Book Co. Inc., New York, 1951, Appendix.

Conclusion 2 - The author indicates that the knowledge of soil mechanics under pile driving is incomplete. No matter how mathematically accurate driving resistance computations may be, they are no better than the assumptions used. Soil conditions at various borings may differ and between borings may differ from those at borings.

Conclusion 3 - The use of Micarta cap blocks is a valuable development. It is understood that a large number of piles can be driven with one block. This avoids considerable rapid changes in energy losses and sets that occur when the frequent replacements of wood blocks are necessary. This disposes of a variation that has generally been ignored and that affected the resistance of a pile during its driving and also the uniformity of comparative results between various piles. More information on this subject would be useful because the use of such cap blocks would seem to be advantageous to the engineer, pile driving contractor, and owner.

Conclusion 4 - The author states that the method may be used to determine the ranges of application through which other formulas may be considered reasonably accurate. To serve as a calibration standard in this way, a body of comparisons of wave equation results with load test capacities should be available to indicate the degree of applicability of the equation. Sufficient cases would be required to prove yardstick limits, but would be far fewer than required for comprehensive tabular or graphical presentations. Load test results, published or unpublished, exist for many piles, with sufficient information to enable these cases to be figured by the wave theory on an electronic computer. This would soon make a body of comparisons available, whereas a longer wait would be required to obtain and compare load test results from future jobs.

Conclusion 5 - The wave equation method can be used to determine compressive and tensile stresses during driving in any type of soil, including cohesive soils if side friction is taken into account. This determination of stresses would also be of value in the case of driving on thin-walled steel and on wood piles.

Conclusion 6 - The author states that when loads are to be heavy and resistances are great, heavy piles are usually easier to drive than light piles. The writers have observed this effect when using the Hiley formula. The elastic-shortening term is much larger for light piles, thus indicating considerable wasted energy, and is low for heavy piles or mandrels. On the other hand, the impact term containing ratios of pile and hammer weights is low for heavy piles and high for light piles. The net result is that, under hard driving, the impact losses from a heavy pile become less than the elastic losses from a light pile.

Comparison of Results.—The wave equation has been presented as a tool for determining driving resistances for all combinations of pile and hammer types and sizes. The author has expressed a hope that the results of many computations of capacities by the proposed method be correlated with results of pile load tests carried to failure. This is essential, and the words "carried to failure" are very important and vital to correlation, although many load tests are made to only one and one-half to two times a selected working load. Such tests fulfill many code requirements, but they are intended only to ensure safety and are not concerned with overdesign or an economic solution.

The degree of deviation, likely or unavoidable, for pile driving resistances, seems somewhat analogous to that of concrete test cylinder failure points. For a specified 28-day strength, these samples are presumed to meet required composition and consistency and to be carefully prepared in a uniform manner. In spite of this care, for a typical particular job, in which a 4,000 psi test value was specified, test values showed a wide distribution between 3,000 psi and 5,000 psi (a range of ± 25%), with a few random tests outside these limits. Soil characteristics often show greater variations than does the concrete in the test cylinders. Similarly, some considerable spread of driving resistances must be expected.

When comparing the spread of results of driving resistance formulas versus load tests, the tendency is to assume the load test result as being correct and to measure driving resistance against this value. The entire deviation probably should not be charged to incorrectness of the formula, considering probable variations in the soil and divergence of results from using the numerous criteria for determining failure points. The time element effects a comparison, because soil setup or relaxation may follow driving; in these cases, the comparison actually obtained is between immediate driving resistance and load carrying capacity after possible soil changes. Redriving might give a more truthful comparison; however, most reported driving resistances have been those at the close of initial driving. A spread of agreement should, as a practical matter, be considered as normal and unavoidable.

If it can be determined that the Hiley or EN formula results are safe and that ultimate driving resistances are in reasonable agreement with wave equation results in any general range of conditions, then such formulas might permissibly be used within such limits. This might enable such simple formulas to be quickly applied in the office or field, thus avoiding the necessity of access to a computer in order to be sure of obtaining sufficiently reliable and economic results. Furthermore, establishment of a computer program for solving resistances by the Hiley equation is quite simple, provided that the volume of work warrants it.

The writers have computed the driving resistances of 80 load-tested piles by the Hiley formula. Effective lengths were full lengths for end-bearing piles and assumed lengths to centers of driving resistances for piles carried partially or entirely by friction. The permanent load-carrying strata in all cases were sand or sand and clay. Test load values have been plotted against driving resistances in Figs. 13 and 14. If these values are identical, the points of intersection should fall on a 45° line. Lines representing a 25% variation each way from the line ofccoincidence form an envelope that contains most cases. This should be satisfactory when using a factor of safety such as 2.5 and would result in a range of 1.9 to 3.1, or say 2 to 3.

Agreement between driving resistances computed by the Hiley formula and load tests appears to the present writer to be satisfactory for the piles shown in Figs. 13 and 14. In a few cases, load-test results were appreciably greater than the Hiley formula results; such cases seem to have been characterized by piles that were both long and heavy, although computed values of some piles both fairly long and heavy were in good agreement with load test values. Computed driving resistances of piles, either long or heavy, but not both, showed good agreement between computed driving resistances and load-test values. The number of cases shown is not enough to state that the above effects will always hold true, but the results are suggestive of possible relationships.

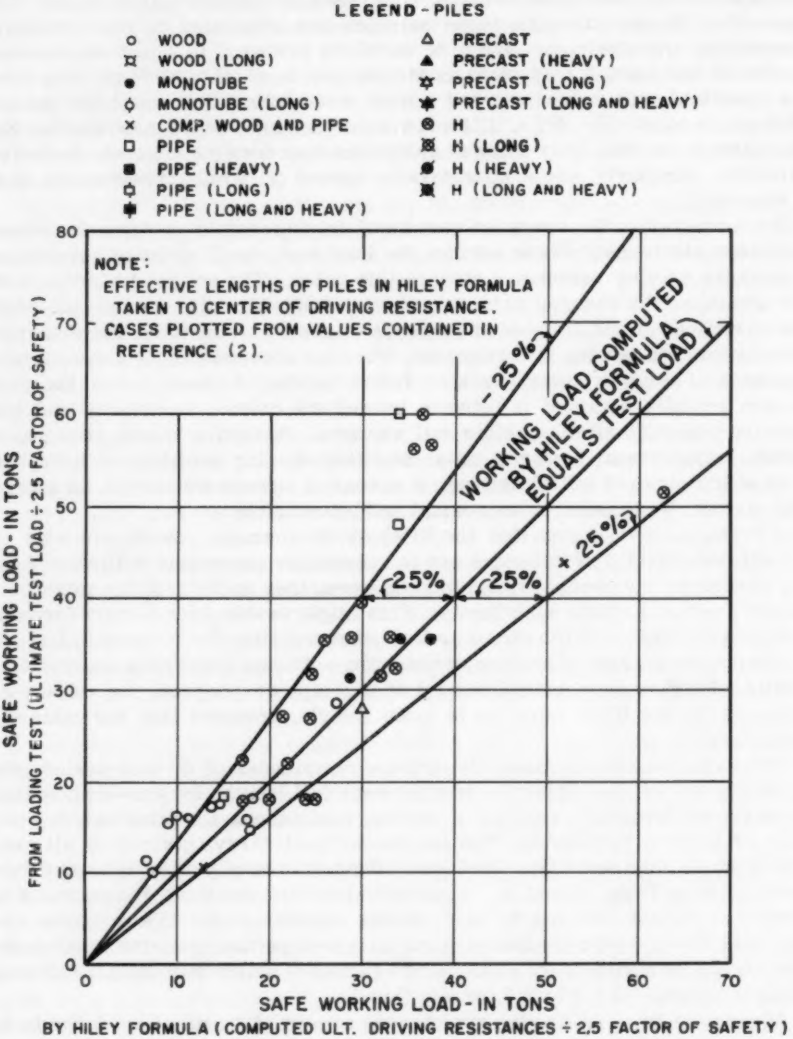

FIG. 13.—HILEY FORMULA AND LOAD TESTS

The Hiley formula and test load results, compared in Figs. 13 and 14, show groupings fairly well averaged above and below the line of exact agreement. Wave equation curves such as shown in Figs. 15 and 16 appear to lie in somewhat higher ranges of driving resistance values than do the Hiley curves. These curves are all for full end-bearing piles. Relatively small driving resistance

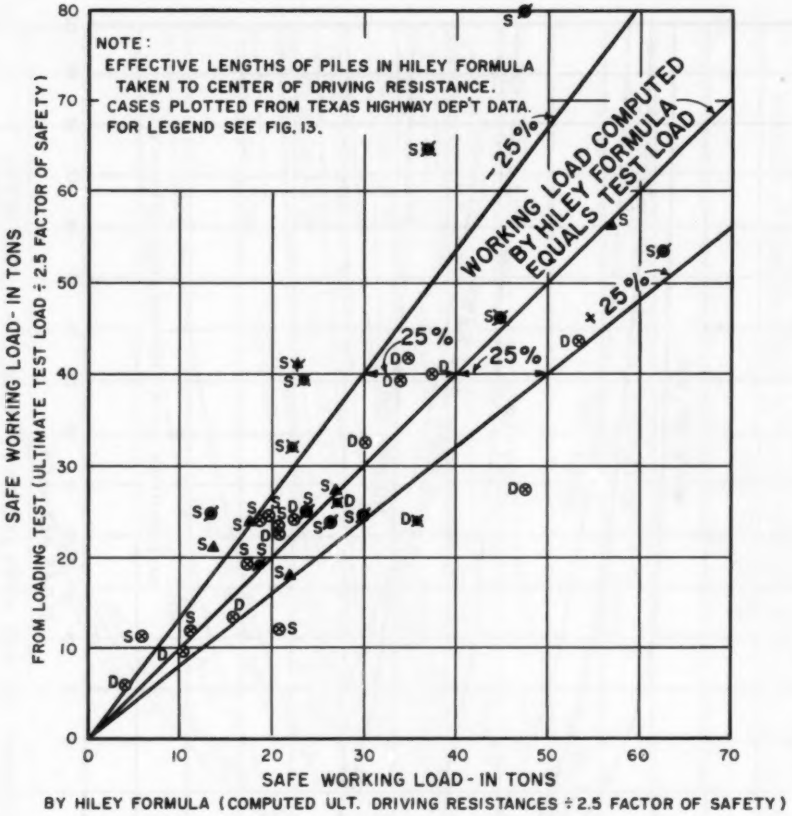

FIG. 14.—HILEY FORMULA AND LOAD TESTS, H PILES AND PRECAST PILES DRIVEN BY DROP HAMMER (D) OR STEAM HAMMER (S)

differences appear between short and long piles and occur when using the wave equation as indicated in Figs. 15 and 16, whereas driving resistances decrease quite materially for longer piles when using the Hiley formula. This tends to indicate that the ranges of wave equation values might be too high, and that lower results would agree better with load tests.

It should be carefully noted, however, that the wave equation curves shown in Figs. 15 and 16 represent the result of calculations made 4 or 5 yr ago before the damping constants J and J' were introduced. Use of these damping constants, as recommended by the author, would give lower values and thus bring the wave equation results into closer agreement with the Hiley formula except probably for piles that are both long and comparatively heavy. At the present time (1961) an electronic computer at the Agricultural and Mechanical College of Texas is being programmed for pile calculations by the wave equation, and in due course the results of these calculations will become available. At that time more accurate comparisons can be made.

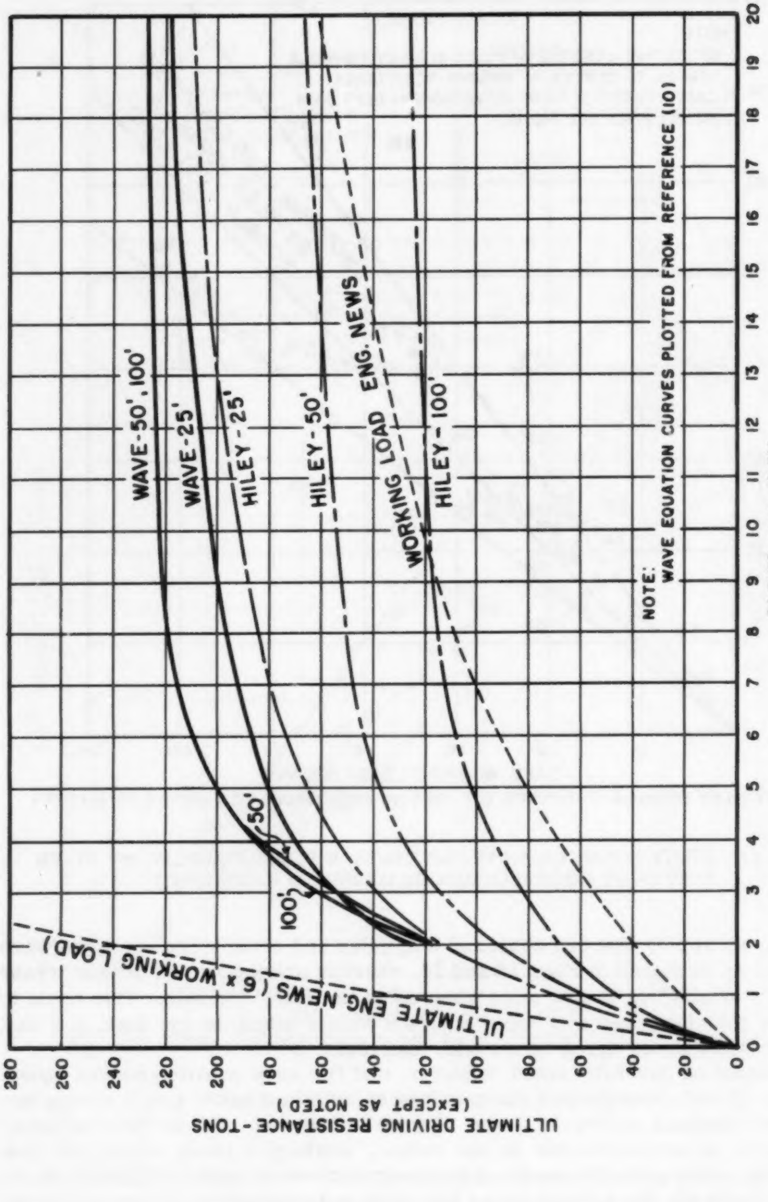

FIG. 15.—WAVE EQUATION AND ENG. NEWS AND HILEY FORMULAS 14 INCH OCTAGONAL PRECAST PILES—NO. 0 VULCAN (SAME FOR 80C) END BEARING

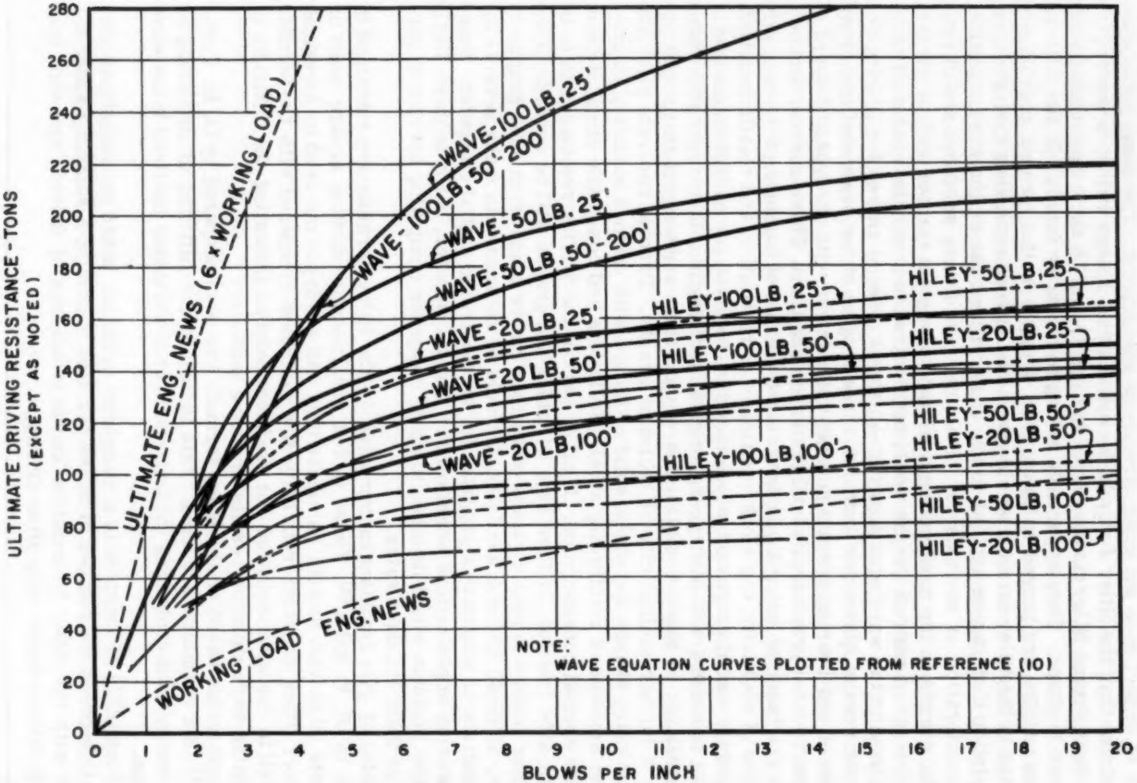

FIG. 16.—WAVE EQUATION AND ENG. NEWS AND HILEY FORMULAS UNIFORM STEEL PILES—NO. 1 VULCAN (SAME FOR 50C) END BEARING

The author mentions another publication[3] as indicating that the Newtonian theory of impact does not apply directly to pile driving. The same publication also states that the Hiley formula includes some losses twice; however, the wave equation and Hiley curves shown in Figs. 15 and 16 show remarkable similarities in shape. They apparently belong to the same family. If the wave equation results are proved to be correct, it might be that proper coefficients inserted in the Hiley formula would give close agreements, thus giving a simple solution that can be applied to any case without use of the wave equation.

Consideration of methods for obtaining wave theory solutions and of presenting results in the most economical and convenient manner will be needed. A very large number of graphs would be required to cover the ranges of variables used in the wave equation. Fig. 15 shows a set of curves for driving resistance versus blows per inch for three lengths of the same uniform cross section of end-bearing precast concrete pile driven with one type and speed of hammer; such a graph applies only to these conditions. These curves, furthermore, are based on using a constant selected value of hammer efficiency, pile modulus of elasticity, cap block coefficient of restitution, and elastic constant. A separate set of graphs or tables would be required for easy combination of driving hammers and pile types, weights, and lengths. Step-taper pile cores are available in many combinations of diameters and weights that would require a further multiplicity of tables or graphs. Precast concrete piles are used in many shapes and sizes, solid or hollow with various moduli of elasticity and amounts of reinforcing steel. Followers and composite piles, if used, require special consideration. If all or part of the driving resistance is to be met in side friction, further sets of tables and graphs will be needed. If the designer wishes to obtain values other than can be found from such graphs or tables, if these become available, it would be necessary to make several interpolations or establish a program for an electronic digital computer. Some engineering organizations might own or rent a computer, but it is more likely that the problem would be taken to some computer computing service to perform at their standard rates.

Tables of pile resistances computed by the Hiley formula are contained in a book by R. V. Allin.[40] These tables are based on driving square wood or concrete piles with British hammers only and are types not used in American practice. They consider relatively few variables compared with the number involved in the wave equation but are mentioned as indicating the difficulty of presenting wave equation results in such a manner.

A nomogram based on the Hiley formula has been prepared by G. M. Cornfield.[41] Six variables are used. This indicates the difficulty of preparing or using nomograms containing the large number of variables involved in the wave equation.

A possibility of interest is a computer program based on statistical correlation between the number of blows per foot in driving a sampling spoon together with the other information known in advance of driving and with load tests to failure known only after driving.

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[40] "The Resistance of Piles to Penetration," by R. V. Allin, E. & F. Spon Ltd., London, 2nd Edition.

[41] "A Direct Reading Hiley-Formula-Based Nomogram for Steel Bearing Piles," by G. M. Cornfield, Civil Engineering and Public Works Review, London, July, August, 1959, pp. 880-881.

Future Procedures.—It seems desirable to explore the place of the proposed method in the entire problem of foundation design.

The proposed theory of longitudinal impact gives only driving resistances of individual piles. It does not consider the value of the supporting soils or the effects of group action any more than do any dynamic driving formulas. This factor alone may serve to make considerable changes in the allowable pile load, but the various methods of computing reductions show a wide range, indicating so large an unknown that great accuracy in predicting pile carrying capacities may not be possible.

In future years, many engineers may become familiar with electronic digital computer programming and use such a method extensively, but for some years at least many pile problems will continue to occur in offices or in the field in which computers or programmers may not be available. Unless, or until, universal acceptance of this method takes place and means for its use are available and practicable for each combination of factors, the degree of reliability of the current methods for all types and sizes of piles and hammers remains of interest.

It is hoped that the subjects of correlation of the wave equation results with test load values and with the Hiley and Engineering News formulas will be carried forward and results publicized, including those for full or partial piles.

SEEPAGE REQUIREMENTS OF FILTERS AND PERVIOUS BASES[a]

Discussion by W. J. Turnbull, Edward S. Barber

W. J. TURNBULL.[19]—Mr. Cedergren is to be commended on the development of rational procedures in designing various types of filter blankets, both sloping and horizontal, to ensure that the filter blanket not only is sufficiently permeable and of such sizing to prevent movement of particles from the base into the filter, but also is of such thickness or so designed that hydrostatic pressure does not build up in the filter itself. The latter, of course, can be accomplished by a proper cross-section areal design of the filter itself and/or a collection system so designed that no backing up of water in the filter is possible.

The charts developed by Mr. Cedergren can, as he points out, be used for designing filters for similar structures, but more important is the fact that they do demonstrate how a filter design problem can be approached.

It is believed that the most important contribution made by Mr. Cedergren's paper is in calling attention to the great importance of properly designed filter drains from the base or medium that is being drained through the filter itself and on through the collection system, with attention being directed to the fact that each phase of the system needs to be properly designed to assure: (a) no piping of base material with resultant loss or plugging of a portion of the filter, (b) no piping of filter material, and adequate permeability and cross-section area of the filter, and (c) proper collection of seep water to eliminate back pressure. Proper gradation of the filter material is a very important part of items (a) and (b).

It is the writer's observation that in some very important structures the design of filters is still rather carelessly accomplished. It has also been noted that many engineers tend to want to economize on the filtering system rather than on some other phase of the structure that is less critical. The encouraging factor, however, is that this tendency among engineers is much less than it was 25 yr or 30 yr ago when it was not unusual to find that practically all filters under or behind hydraulic structures never functioned properly. In fact it is recalled that one prominent engineer in those days remarked that it was the popular thing to put filters under the floors and behind the walls of hydraulic structures but that he did not expect them to work; rather, he wanted enough overdesign in the structure to ensure the safety of the structure on the assumption that the filters would not function properly.

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[a] October, 1960, by Harry R. Cedergren (Proc. paper 2623).

[19] Engr., Chf. of Soils Div., U. S. Army Engr. Waterways Experiment Station, Vicksburg, Miss.

EDWARD S. BARBER,[20] A. M. ASCE.—The paper provides useful solutions of two practical problems and properly emphasizes the importance of boundary conditions and permeability on drainage capacity.

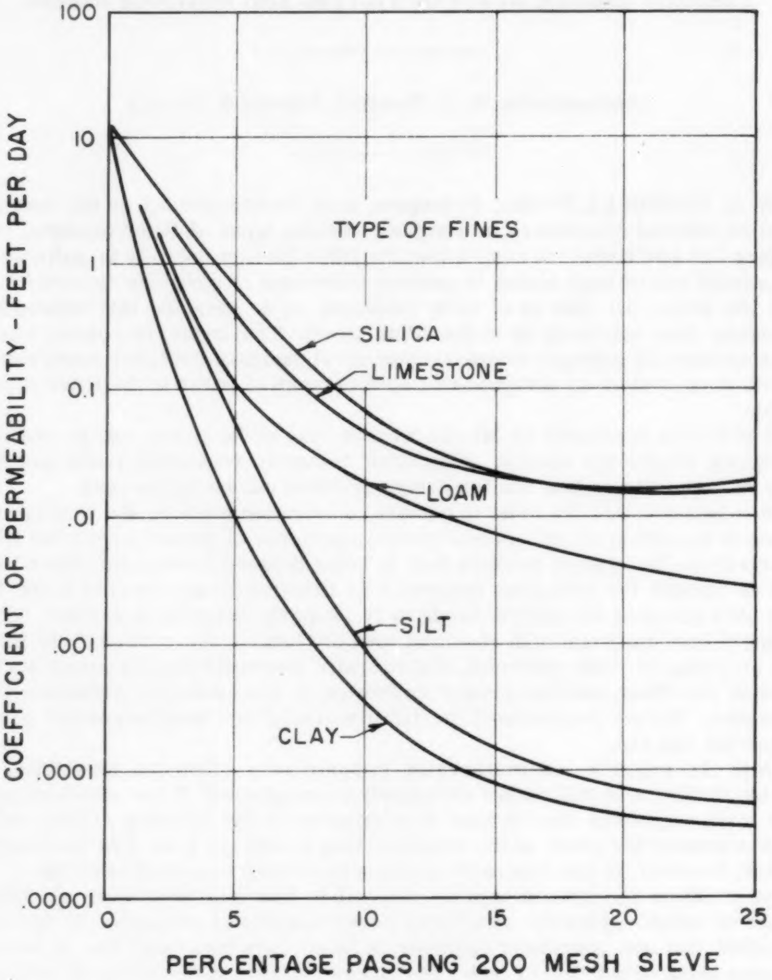

FIG. 10.—EFFECT OF FINES ON PERMEABILITY OF GRADED AGGREGATE

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As a further example, consider a layer of permeability, k, thickness, h, and width, b, on an impervious base. Its capacity to transmit uniform infiltration to one side[21] is

$$q = k\left(\frac{h}{b}\right)^2 \qquad (2)$$

With k = 100 ft per day, vertical transmission with no obstruction would be 100 cu ft per sq ft per day; but, with h = 10 ft and b = 5,000 ft, $q = 100\ (10/5{,}000)^2 = 0.0004$ ft per day or 1.8 in. per yr. Such a condition necessitated subsurface drainage at Idlewild Airport in clean sand because the natural transmission was less than the annual rainfall. Similarly, for low ratios of h to be in highway base courses, very pervious materials are required to transmit laterally appreciable quantities of infiltration. Taking q as the maximum rate of settlement, Eq. 2 can be used to determine the required permeability or thickness of a sand blanket over vertical sand drains.

Fig. 10 shows how a small amount of fines, if uniformly dispersed, will drastically reduce the permeability of clean granular material.

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[21] "Field Measurements for Tests of Soil Drainage Theory," by D. Kirkham and J. W. deZeeuw, Proceedings, Soil Science Society of America, Vol. 16, 1952, pp. 286-293.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 2703 is identified as 2703(ST1) which indicates that the paper is contained in the first issue of the Journal of the Structural Division during 1961.

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FEBRUARY: 2355(CO1), 2356(CO1), 2357(CO1), 2358(CO1), 2359(CO1), 2360(CO1), 2361(PO1), 2362(HY2), 2363(ST2), 2364(HY2), 2365(SU1), 2366(HY2), 2367(SU1), 2368(SM1), 2369(HY2), 2370(SU1), 2371(HY2), 2372(PO1), 2373(SM1), 2374(HY2), 2375(PO1), 2376(HY2), 2377(CO1)[c], 2378(SU1), 2379(SU1), 2380(SU1), 2381(HY2)[c], 2382(ST2), 2383(SU1), 2384(ST2), 2385(SU1)[c], 2386(SU1), 2387(SU1), 2388(SU1), 2389(SM1), 2390(ST2)[c], 2391(SM1)[c], 2392(PO1)[c].

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APRIL: 2428(ST4), 2429(HY4), 2430(PO2), 2431(SM2), 2432(PO2), 2433(ST4), 2434(EM2), 2435(PO2), 2436 (ST4), 2437(ST4), 2438(HY4), 2439(EM2), 2440(EM2), 2441(ST4), 2442(SM2), 2443(HY4), 2444(ST4), 2445 (EM2), 2446(ST4), 2447(EM2), 2448(SM2), 2449(HY4), 2450(ST4), 2451(HY4), 2452(HY4), 2453(EM2), 2454 (EM2), 2455(EM2)[c], 2456(HY4)[c], 2457(PO2)[c], 2458(ST4)[c] 2459(SM2)[c].

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FEBRUARY: 2726(WW1), 2727(EM1), 2728(EM1), 2729(WW1), 2730(WW1), 2731(EM1), 2732(SM1), 2733(WW1), 2734(SM1), 2735(EM1), 2736(EM1), 2737(PL1), 2738(PL1), 2739(PL1), 2740(PL1), 2741(EM1), 2742(ST2), 2743(EM1), 2744(WW1), 2745(WW1), 2746(SM1), 2747(WW1), 2748(EM1), 2749(WW1), 2750(WW1)[c], 2751 (EM1)[c], 2752(SM1)[c], 2753(PL1)[c], 2754(ST2)[c], 2755(PL1).

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[c] Discussion of several papers, grouped by divisions.

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